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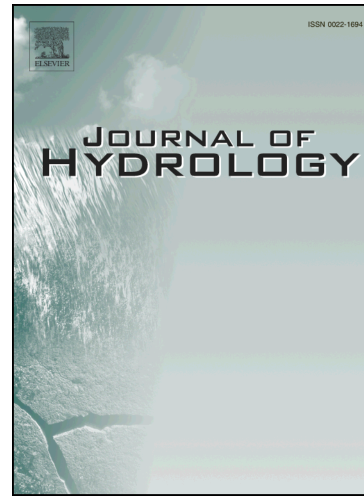

Research papers

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Joint modelling of annual maximum drought severity and corresponding duration

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ABSTRACT

In recent years, the joint distribution properties of drought characteristics (e.g. severity, duration and intensity) have been widely evaluated using copulas. However, history of copulas in modelling drought characteristics obtained from streamflow data is still short, especially in semi-arid regions, such as Turkey. In this study, unlike previous studies, drought events are characterized by annual maximum severity (AMS) and corresponding duration (CD) which are extracted from daily streamflow of the seven gauge stations located in Çoruh Basin, Turkey. On evaluation of the various univariate distributions, the Exponential, Weibull and Logistic distributions are identified as marginal distributions for the AMS and CD series. Archimedean copulas, namely Ali-Mikhail-Haq, Clayton, Frank and Gumbel-Hougaard, are then employed to model joint distribution of the AMS and CD series. With respect to the Anderson Darling and Cramér-von Mises statistical tests and the tail dependence assessment, Gumbel-Hougaard copula is identified as the most suitable model for joint modelling of the AMS and CD series at each station. Furthermore, the developed Gumbel-Hougaard copulas are used to derive the conditional and joint return periods which can be useful for designing and management of reservoirs in the basin.

Keywords: Drought, Annual maximum severity, Corresponding duration, Archimedean copulas, Çoruh Basin, Turkey

1. Introduction

Drought is a complex phenomenon that can be mainly characterized by its severity and duration. These characteristics are usually derived from hydro-meteorological data, such as precipitation, streamflow and groundwater, using the threshold level approach. Among these data, if the main parameter of interest is streamflow, it is defined as hydrologic drought. The severity and duration characteristics of the hydrological drought are the main concern of engineers and hydrologists for design, planning and management of water resources structure. It should be noted that studies of drought duration and severity have great importance for hydropower installations and reservoir policies. Water resources managers and engineers utilize joint and conditional return periods of drought duration and severity as a hydraulic design criterion and give valuable information for assessing hazard (Mirabbasi et al., 2012; Shiau, 2006). Since the drought severity and duration characteristics are random variables, they are analyzed and modeled using probabilistic theories. Approaches to probabilistic analysis of droughts are univariate or multivariate. Drought severity and duration are also mutually correlated variables, hence, the univariate analysis of the drought characteristics is inadequate to account the significant correlation between the variables. Also, in design and management of water supply systems, it is not enough to know information about drought duration only, but it is also essential to estimate severity value of that duration. Therefore, in recent years, multivariate analysis of drought events has attracted more interest than univariate analysis. For this purpose, traditional multivariate distributions (such as multivariate normal, multivariate lognormal, multivariate weibull etc.) have been employed by a number of researchers (Nadarajah, 2007; Nadarajah, 2008; Nadarajah, 2009; Yue et al., 2001). However, these distributions usually suffer from several limitations and constraints. For example, individual behavior of drought variables must be characterized by the same parametric family of univariate distributions. During last decade, copulas have emerged as a new multivariate method to overcome such difficulties. Copulas can both preserve dependence structure and different distribution characteristics of the random variables. Therefore, the copulas have recently gained popularity in multivariate modelling of drought characteristics. Multivariate modelling of hydrological drought is also necessary in the reservoir design and management. Crisis-oriented drought response efforts have been largely ineffective, inadequately coordinated, unfavorable, and inefficient as far as the allocated resources. One approach to make management of drought easier is to build up an arrangement of progressively strict conservation measures in view of a sequence of drought triggers, and look for public's approval. The present water management condition in the basin (Çoruh Basin) is not really planned with a numerousness of projects and initiatives under the auspices of unique regions, and a diversity of private sector performers and global funding organizations. Expanded cooperation and offer of information on developing drought occasions at basin level can lessen possible risks (Awass, 2009). In the majority of previous works on copula based multivariate drought modelling, drought characteristics have been extracted from precipitation data (Chen et al., 2013a; Ganguli and Reddy, 2012; Lee et al., 2013; Ma et al., 2013; Mirabbasi et al., 2012; Mirakbari et al., 2010; Rauf and Zeephongsekul, 2014; Reddy and Ganguli, 2012; Shiau, 2006; Shiau and Modarres, 2009; Song and Singh, 2010; Tosunoglu and Can, 2016; Yoo et al., 2013; Yusof et al., 2013; Zin et al., 2013) and a few attempts have been done on joint modelling of drought characteristics obtained from streamflow data. Among these, Shiau et al. (2007) used bivariate copula functions to build joint distributions of drought severity and duration series obtained from the monthly streamflow in Yellow River basin, China. Sadri and Burn (2014) used

Archimedean family of copulas, namely Clayton, Frank and Gumbel-Hougaard, for joint modelling of drought duration and severity series extracted from monthly streamflow from 36 non-regulated sites in the Canadian Prairies (see also Chen et al., 2013b; Kwak et al., 2014; Zhang et al., 2013). Also, to the best knowledge of the authors of this article, there is not any published study in the literature related to application of copulas for joint modeling of maximum drought characteristics. In addition, the copula based drought analysis has not been previously employed for the Çoruh Basin which has a number of dams and hydroelectric power plants where the studies of drought duration and severity have great importance. Hence, the present study aims to model the joint distributions of annual maximum drought severity and corresponding maximum drought duration series obtained from streamflows. Threshold level method is used to define maximum severity and duration series of daily streamflow data obtained from seven gauge stations located in the Çoruh Basin, Turkey. Marginal univariate distributions of the derived drought characteristics are then determined. Furthermore, Archimedean families of copulas, namely Ali-Mikhail-Haq, Clayton, Frank and Gumbel-Hougaard copulas, are employed to model joint distribution function of the derived drought characteristics. The most appropriate copulas are then used to derive conditional and joint return periods of drought characteristics, which can be useful for designing and management of reservoirs (e.g. for hydropower, drinking and irrigation water supply) in the basin.

2. Study area and data

The study area Çoruh Basin has an area of 19,748 km^2, which is approximately 2.53% of Turkey, located in North-East Turkey. Because of prevailing climate conditions and geological characteristics over different parts of the basin, it can be defined as three main sub-basins, namely lower, middle and upper. The basin's average annual rainfall is about 480 mm whereas average rainfall over the year in Turkey is around 642 mm (Yerdelen et al., 2010). The Çoruh River is about 410 km that is the longest river of the East Black Sea region and the river has the third highest runoff coefficients of 26 basin rivers in Turkey. The Çoruh Basin and its rivers have a high economic importance to Turkey because it is largely undeveloped but has economically exploitable hydropower potential (Berkun, 2010). In order to use the energy potential of the river, a number of dams and hydroelectric power plants (HEPP) have been recently designed; the construction of some is already completed while some are under construction or in the project phase. Location of the Dams and HEPPs is presented in Figure 1. Among these, Deriner Dam and HEPP is the most important scheme of the lower part development of the Çoruh River. The Deriner Dam whose reservoir began to fill in 2012 and its power station was completed in 2013. It is the highest dam of any kind in Turkey and ranks in the top 10 of the highest concrete dams in the world with a height of 253 m. It is expected to generate 2118 GWh of electrical energy annually. In this study, daily streamflow records from seven gauge stations were selected for hydrologic drought analysis and copula modelling (Figure 1). All stations are operated by General Directorate of State Hydraulic Works, Turkey. The record lengths vary from 28 to 45 years (see Table 2). Observations are not affected by water reservoirs since the considered data periods include the time before the reservoirs were constructed. More detailed information (e.g., type of dam, construction time period, location, power, etc.) of these reservoirs can be found in Akpinar et al. (2011).

Figure 1

3. Methods

3.1. Definition of annual maximum drought severity and corresponding duration

The threshold level method is most frequently applied to define hydrologic drought events from streamflow time series. A drought event is said to be a period which the flow is below the threshold. Therefore, the most important decision for hydrologic drought definition is the selection of the threshold value. There are various levels that have been used as threshold value in hydrologic applications. However, the widespread threshold types are the median or mean flow (Shiau et al. 2007) and the flow values equal to 70 (Q_{70}) and 90 (Q_{90}) % of the time from flow duration curves (Edossa et al., 2010; Hisdal et al., 2004). After defining drought events according to the selected threshold value, important drought parameters, such as duration (D) and severity (S) are easily determined. Drought duration is the length of period in which the hydrological variable values (in here daily streamflow) are less than truncation value, drought severity is the cumulative streamflow (the total deficit) value based on the duration time. Annual maximum drought severity (AMS) is the largest value of the computed severity series for each year and corresponding duration (CD) is length of maximum drought severity. Figure 2 illustrates the definition of drought characteristics extracted from threshold level method. In this study, threshold level Q_{70} values are selected to investigate the streamflow drought events at the considered stations.

Figure 2

3.2. Univariate marginal distributions for drought characteristics

Definition of marginal distribution for drought characteristics is essential step before launching into a multivariate model. Based on different characteristics of droughts to be examined, various univariate distributions have been employed in many studies. In this study, eleven probability distributions usually applied in hydrological frequency analysis are considered. The distributions are Exponential, Extreme Value, Weibull, Gamma, Generalized Extreme Value, Generalized Pareto, Inverse Gaussian, Logistic, log-Logistic, Normal and Lognormal distributions. To define the most suitable type among alternative distributions, Akaike Information Criterion (AIC) developed by Akaike (1974) is used and it can be expressed as;

$$AIC = -2 \cdot logL + 2 \cdot k \tag{1}$$

where log L is the log likelihood of the model and k denotes the number of model parameters. The most appropriate distribution is the one which has the minimum AIC value.

3.3. Joint distribution of drought characteristics: Copulas

To obtain joint distribution of random variables that follow different distributions, copulas, which have been proposed by Sklar (1959), are recently used as a powerful and relatively new technique in the field of hydrology. Major advantage of copulas lies in modelling the dependence structure of the univariate marginal distributions independently. Thus, this gives great freedom to choose the univariate marginal distributions. Considering a situation with two random variables, according to Sklar's Theorem if $F_{X,Y}(x,y)$ is a two-dimensional cumulative distribution function (cdf) with marginal cdfs $F_X(x)$ and $F_Y(y)$ then there exists a copula *C* such that

$$F_{X,Y}(x,y) = C\big(F_X(x), F_Y(y)\big) \tag{2}$$

Conversely, assuming that $F_X(x)$ and $F_Y(y)$ are cdfs of univariate distributions of random variables and C is any type of copula, the $F_{X,Y}(x,y)$ then becomes a two-dimensional distribution function with marginal distributions $F_X(x)$ and $F_Y(y)$. Furthermore, if $F_X(x)$ and $F_Y(y)$ are continuous, then *C* is unique (Shiau, 2006). Under the assumption that the marginal distributions are continuous with probability density functions $f_X(x)$ and $f_Y(y)$ the joint probability density function then becomes

$$f_{X,Y}(x,y) = c\big(F_X(x), F_Y(y)\big) f_X(x) f_Y(y) \tag{3}$$

where *c* is the density function of *C*, defined as

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{4}$$

In which, $u = F_X(x)$ and $v = F_Y(y)$.

3.4. Archimedean copulas

In this study, the Archimedean class of copulas, namely the Ali-Mikhail-Haq family, the Clayton family, the Frank family, and the Gumbel-Hougaard family are considered due to the fact that these copulas are easy to construct and they can capture wide ranges of dependences (e.g. Chang et al., 2016; Grimaldi and Serinaldi, 2006; Lee et al., 2013; Li et al., 2013; Reddy and Ganguli, 2012; Sadri and Burn, 2014; Shiau et al., 2007; Sraj et al., 2015; Wang et al., 2009; Wong et al., 2010; Zhang and Singh, 2006; Zhang and Singh, 2007). According to Nelsen (2006), the n-dimensional Archimedean copula can be expressed as:

$$C(u_1, u_2, \dots \dots \dots \dots \dots \dots .., u_n) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2) \dots \dots \dots \dots \dots . + \varphi(u_n)) \quad (5)$$

where φ is the copula generating function and $u_1, u_2, \dots\dots u_n$ are cumulative distribution functions (CDFs) of univariate distributions of random variables. Two dimensional Archimedean copula is then expressed as;

$$C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)) \quad (6)$$

φ^{-1} is the inverse of the generating function φ. Mathematical expressions of the selected copulas and their generating functions are summarized in Table 1.

Table 1

3.5. Goodness of fit tests for bivariate copulas

Once the characteristics of a set of copulas have been estimated, the next step in the process of fitting copulas to empirical data consists of selecting the most suitable copula among the various copulas under consideration. For this context, the Akaike information criterion (AIC), the Bayesian information criterion (BIC), the Kolmogorov-Smirnov (KS), the Anderson-Darling (AD), and Integrated Anderson-Darling (IAD) tests have recently been used by a number of researchers. In this paper, an integrated version of the Anderson-Darling test (IAD), which can be more sensible to reduce the impact of outliers, is applied to define the most appropriate copulas. The IAD test statistics for *d* dimensional copula is calculated as follows:

$$D_{IAD} = \sum_{i=i}^{n}\sum_{j=1}^{n}, \ldots\ldots, \sum_{k=1}^{n} \frac{\left(C_n\left(u_{1,i}, u_{2,j}, \ldots\ldots, u_{d,k}\right) - C_\theta\left(u_{1,i}, u_{2,j}, \ldots\ldots, u_{d,k}\right)\right)^2}{C_\theta\left(u_{1,i}, u_{2,j}, \ldots\ldots, u_{d,k}\right) \cdot \left(1 - C_\theta\left(u_{1,i}, u_{2,j}, \ldots\ldots, u_{d,k}\right)\right)} \tag{7}$$

where C_θ represents the parametric copula and C_n denotes the empirical copula calculated from n observational data as follows

$$C_n(u_1, u_2, \ldots\ldots, u_d) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{I}\left(\frac{R_i}{n+1} \le u_1, \frac{S_i}{n+1} \le u_2, \ldots\ldots, \frac{T_i}{n+1} \le u_d\right) \tag{8}$$

where R_i, S_i and T_i denote the ranks of observed data and I is an indicator function taking the value 1 if the condition is satisfied and 0 otherwise. The bivariate copula family with the minimum value of test statistics is selected as the most appropriate model. Parameters of copulas are estimated using three procedures; (1) Method of moments (MOM), (2) Maximum pseudo likelihood estimator (MPLE) and (3) Inference from Margins (IFM). In this study, MOM is used to estimate parameters of the candidate copulas because this method is quite popular in Archimedean family of copulas (Nazemi and Elshorbagy, 2012). Moreover, Cramér-von Mises test, which is one of the most powerful goodness of fit test, is employed to evaluate performance of the copulas. According to Genest et al. (2009), the Cramér-von Mises test statistic, S_n, can be computed as follows;

$$S_n = n\int_{[0,1]^2} \{C_n(u,v) - C_\theta(u,v)\}^2 \mathrm{d}C_n(u,v) = \sum_{i=1}^{n}\left\{C_n\left(U_{i,n}, V_{i,n}\right) - C_\theta\left(U_{i,n}, V_{i,n}\right)\right\}^2 \tag{9}$$

An approximate p-value corresponding to the above test statistic can be obtained via large simulated samples by means of the parametric bootstrapping procedure defined by Genest and Remillard (2008) or by the multiplier approach recently shown by Kojadinovic et al. (2011). Calculation of the p value is;

$$p = \frac{1}{N}\sum_{t=1}^{N} 1\left(S_{n,t} \geq S_n\right) \tag{10}$$

Where N is the number of simulations and the p value is utilized for acceptance or rejection of considered copulas. If the computed p value is larger than a significance level (α), then the copula is accepted as a suitable model (Abdi et al., 2016; Requena et al., 2013). In this study, the p values for each copula are estimated using the parametric bootstrapping procedure and all calculations are carried out with the free software R (R, 2012) and the commercial software Matlab (Matlab 2009a, The Math-Works, Inc.).

3.6. Assessment of tail dependence

Another crucial concern in multivariate frequency analysis of extreme hydrological events, such as droughts and floods, is the tail dependence. If the tail dependence structure amongst drought (or flood) characteristics is not well preserved by chosen copula, it may provide a high uncertainty in estimation of extreme quantiles which consequently causes to inaccurate decisions for design of hydrologic structure. Hence, this evaluation plays an important role in evaluating the adequacy of the selected copula family. For bivariate copulas, tail dependence can be calculated as follows (Poulin et al., 2007);

$$\lambda_U = \lim_{t \to 1^-} \frac{1 - 2t + C_\theta(t,t)}{1 - t} \tag{11}$$

$$\lambda_L = \lim_{t \to 0^+} \frac{C_\theta(t,t)}{t} \tag{12}$$

In which, λ_U and λ_L are the upper and the lower tail dependence, respectively. In analysis of extreme events, the upper tail dependence has greater interest than lower tail dependence. Therefore, we only focus the upper tail dependence in the present study. Several coefficients have been suggested for non-parametric estimation of the upper tail dependence (Serinaldi et al., 2015). In this study, the estimator suggested by Caperaa et al. (1997) and (Frahm et al., 2005) is used to compute nonparametric upper tail dependence coefficient because of its popularity related to its advantages (see Poulin et. al., 2007). This estimator can be estimated based on the following equation;

$$\lambda_U^{CFG} = 2 - 2 \cdot exp\left(\frac{1}{n}\sum_{i=1}^{n} log\left(\frac{\sqrt{\log\left(\frac{1}{u_i}\right) \cdot \log\left(\frac{1}{v_i}\right)}}{log\left(\frac{1}{\max(u_i, v_i)^2}\right)}\right)\right) \tag{13}$$

Where, u_i and v_i are the CDFs of the drought characteristics considered in this work. Copula based upper tail (λ_U) and empirical upper tail (λ_U^{CFG}) coefficients are compared to verify the adequacy of the model. Moreover, it is important to recognize that the Ali-Mikhail-Haq, Clayton and Frank copulas do not have upper tail dependence $(\lambda_U = 0)$ while the Gumbel-Hougaard copula has strong upper tail dependence which can be easily calculated as $\lambda_U = 2 - 2^{1/\theta}$.

4. Results

4.1. Marginal probability distributions of AMS and CD series

As mentioned in the section 3.1, the AMS and CD series for each station were computed using constant threshold level (Q_{70}) that obtained from flow duration curves. The basic statistics properties of the daily streamflow and the computed AMS and CD series are presented in Table 2. In this table, $\bar{X}$ and SD indicate the mean and standard deviation for daily streamflow data and $\bar{X}^{*}$ and SD^{*} are the mean and standard deviation of the drought characteristics. It is apparent from the table that the station 2315 has the lowest variation (SD/$\bar{X}$) while the highest variation belongs to 2325. The highest variation of 2325 may be due to the small drainage area and high elevation of this station and low variation of 2315 is also related to its big drainage area and low elevation. The variation of drought characteristics is lower when compared to daily streamflow data. In conventional frequency analysis, data series under study should be stationary. Therefore, before fitting any univariate distribution to data, the stationarity must be provided by using some statistical methods, such as time trend analysis (e.g. trend test , Spearman'r Rho test, autocorrelation function analysis), time-frequency analysis (e.g.Wavelet analysis*)* and frequency domain analysis (e.g. spectral analysis) (Parmar and Bhardwaj, 2015; Zhang, 2005). In this study, Mann-Kendall test is applied for checking stationarity of the AMS and CD series. The results are summarized in Table 2. In the Mann-Kendall test, the null hypothesis (H_0) is an assumption that the AMS and CD series are stationary at significant level of 0.05. According to Table 2, it can be inferred that there is not significant trend detected by the Mann-Kendall test and hence, the AMS and CD series can be considered as stationary time series. A detailed description of the Mann-Kendall test can be found in (Cigizoglu et al., 2005; Kahya and Kalayci, 2004; Kisi, 2015; Kisi and Ay, 2014; Onoz and Bayazit, 2003; Partal and Kahya, 2006). It should be noted that pre-whitening is not applied here because the AMS and CD series have not serial autocorrelation (see Figure 3a and 3b).

Table 2

Figure 3a and 3b

After the stationarity of the AMS and CD series is provided, estimation of univariate marginal distributions of these series is performed. For this purpose, various univariate probability distributions mentioned in the previous section have been evaluated by the Akaike Information Criterion (AIC). The Maximum Likelihood Method (MLM) was used to estimate the parameters of the selected distributions as it would provide the smallest sampling variance of the estimated parameters, and hence of the estimated quantiles, compared with other methods (Can and Tosunoglu, 2013). The results are represented in Table 3, which shows the most suitable distributions for the AMS and CD series of each station. According to the Table 3, the AIC test results indicate that Exponential and Weibull are the best fit distributions for the AMS series while Weibull and Logistic distributions performed well for the CD series. Probability distribution functions (PDF) of these distributions are also given in Table 3. Moreover, the visual comparison between empirical and theoretical cumulative distribution functions (CDFs) for the AMS and CD series at each station is presented in Figure 4a and 4b. Here, the Hosking's (Hosking, 1990) plotting position formula, $P(X \leq x_i) = \frac{i-0.35}{n}$, was used to calculate empirical cumulative probability. Here, *i* denotes the rank of the observations in ascending order and n is the sample size. The derived theoretical CDFs show good agreement with the empirical CDFs.

Table 3

Figure 4a and 4b.

4.2. Application of bivariate copulas

Prior to fitting the bivariate copulas, it is important to examine dependence structure between the AMS and CD series. In this study, Pearson's (ρ) and Kendall's (τ) correlation coefficients were applied for achieving this goal. The calculated correlation values and their corresponding p values are presented in Table 4. The statistical significance of the correlation values is checked by Student's t test at the significance level of 0.05. The test results indicate that there is a statistically significant positive dependence between the drought characteristics for all stations. However, the Pearson coefficient only represent linear dependence and therefore it may not be useful for heavy-tailed variables. It can be strongly affected by outliers. On the other hand, the Kendall (τ) can describe a wider class of dependencies and shows resistance to outliers (Klein et al., 2011). Hence, the Kendal's correlation might be more suitable in describing dependence structure in this study. Since there was significant positive association between the drought characteristics and they are well fitted by different distributions, copula functions are employed to model the joint distribution. Four types of Archimedean copulas, namely Ali-Mikhail Haq, Clayton, Frank and Gumbel-Hougaard, are fitted and compared. Since the Kendall τ of the Ali-Mikhail-Haq copula needs to be within the range of $[-0.1817, 0.3333]$ this copula was automatically excluded from further analysis. Furthermore, parameters of the three copula models have been estimated using method of moments. The Integrated Anderson-Darling test statistics (D_{IAD}) have been calculated for each copula and the results are given in Table 5. In addition, the performance of each copula have been evaluated by means of the Cramér-von Mises test. For this purpose, the test statistics S_n and its associated p values have been computed from 10,000 parametric bootstrap samples with the same length as the historical data (Table 5). According to the D_{IAD} and S_n statistics, Gumbel-Hougaard copulas having minimum test values perform better than other options. It can be also seen that all the the Gumbel-Hougaard copulas are satisfied with all p-values larger than 0.05. The parameters of the Gumbel-Hougaard copulas are also given in Table 5. For example, the joint cumulative distribution function (CDF) of the most suitable copula for the station 2305 is given by;

$$C(u,v) = exp\left\{-[(-ln\ (u))^{3.893} + (-\ ln(v))^{3.893}]^{1/_{3.893}}\right\} \quad (14)$$

Where *u* and *v* are the CDFs of the Exponential and the Weibull distributions, respectively. Copula based joint CDFs can be obtained in the same way for other stations. Having selected the Gumbel-Hougaard as the most suitable copula for all stations, nonparametric and parametric values of upper tail dependence coefficient have been calculated to assess the sufficiency of the copula model. The computed values are presented in Table 6. From the table, it can be seen that the Gumbel-Hougaard copulas provided good estimation of upper tail limit compared to the empirical ones although there were some minor differences. Moreover, as a visual evaluation, 10,000 pairs were generated by the derived Gumbel-Hougaard copulas and compared with historical data. The comparison scatter plots of generated data (grey points) with overlapped historical samples (red points) are illustrated in Figure 5. From these plots, it can be observed that the Gumbel-Hougaard copulas perform satisfactorily since the generated data are adequately overlapped with the natural dependence of historical data. As a result, the adequacy of the Gumbel-Hougaard copulas has been proven and allow us to use them for further analysis.

Table 4

Table 5

Table 6

Figure 5

4.3. Bivariate return periods of the AMS and CD series

Estimation of the return periods of drought characteristics is one of the fundamental subject in the planning and management of water resources systems. Univariate return periods of the considered drought characteristics can be estimated by;

$$T_{AMS} = \frac{E(L)}{1-F_{AMS}(ams)} \qquad and \quad T_{CD} = \frac{E(L)}{1-F_{CD}(cd)} \tag{15}$$

In these equations, T_{AMS} and T_{CD} are the return periods with an annual maximum drought severity (or corresponding duration) greater than or equal to a certain value *ams* (or *cd*). $F_{AMS}(ams)$ and $F_{CD}(cd)$ are the cumulative distribution function of annual maximum drought severity and corresponding duration series, respectively. E(L) is the expected drought inter arrival time. Here, the E(L) value is equal to 1 year as annual maximum drought characteristics are considered. However, as mentioned earlier, droughts are multivariate events characterized by mutually correlated variables and univariate analyses of these variables may not be sufficient for assessment and management of droughts. In other words, the return period studies that consider univariate cases may lead to an under or overestimation of the risk (Salvadori and De Michele, 2007). Bivariate return periods can be easily derived using copula based joint distribution function. These return periods can be derived in two ways. One is the joint return periods for drought characteristics and the other one is the conditional return periods for drought characteristics. The joint drought duration and severity return periods are defined for two cases: the return period for *AMS ≥ ams* and *CD ≥ cd*; and the return period for *AMS ≥ ams* or *CD ≥ cd*; These joint return periods for copula based drought events are denoted by $\hat{T}_{AMS,CD}$ and $\check{T}_{AMS,CD}$ respectively as follows;

$$\hat{T}_{AMS,CD} = \frac{1}{1 - F_{AMS}(ams) - F_{CD}(cd) + F_{AMS,CD}(ams,cd)} = \frac{1}{1 - F_{AMS}(ams) - F_{CD}(cd) + C\big(F_{AMS}(ams), F_{CD}(cd)\big)} \tag{16}$$

$$\check{T}_{AMS,CD} = \frac{1}{1 - F_{AMS,CD}(ams,cd)} = \frac{1}{1 - C\big(F_{AMS}(ams), F_{CD}(cd)\big)} \tag{17}$$

Where, $C\big(F_{AMS}(ams), F_{CD}(cd)\big)$ indicates copula based joint distribution function of the drought characteristics. Using the derived bivariate copula functions for each station, the joint return periods of the AMS and CD series were computed and contours of the different return periods (2, 3, 5, 10, 20, 50, 100, 200 and 500 years) of the AMS and CD series are presented in Figure 6. Here, since different combinations of the correlated AMS and CD variables can occur in the same period, the return periods are shown using the contour lines. Historical drought events are also included in the graphs. From these graphs, the joint return periods of the historical drought events can be easily analyzed. For example, the most severe drought event for the station 2315 was appeared in 1966, with an annual severity of 4555 m^3/s days and corresponding duration of 149 days and the joint probability ($\hat{T}_{AMS,CD}$) of this event is more than 500 years. The reason of this drought may be the human factors (e.g., excessive water withdrawals). In addition the joint return periods, using bivariate copula functions, the conditional return periods, $T_{CD\,|\,AMS \geq ams}$ and $T_{AMS\,|\,CD \geq cd}$ can be obtained using following equations;

$$T_{CD\,|\,AMS \geq ams} = \frac{1}{(1 - F_{AMS}(ams)) \cdot (1 - F_{CD}(cd) - F_{AMS}(ams) + C(F_{AMS}(ams), F_{CD}(cd))} \quad (18)$$

$$T_{AMS\,|\,CD \geq cd} = \frac{1}{(1 - F_{CD}(cd)) \cdot (1 - F_{CD}(cd) - F_{AMS}(ams) + C(F_{AMS}(ams), F_{CD}(cd))} \quad (19)$$

The conditional return periods of the CD given various percentile values of the AMS, ($T_{CD\,|\,AMS \geq ams}$) and the conditional return periods of the AMS given various percentile values of the CD, ($T_{AMS\,|\,CD \geq cd}$) were calculated by using Equation 18 and 19. The graphical representations of $T_{CD\,|\,AMS \geq ams}$ and $T_{AMS\,|\,CD \geq cd}$ are provided in Figure 7. Here, due to constrain the paper's length, three examples of the stations (2315, 2316 and 2325) are only given. From these graphs, it can be easily noticed that the graphs indicate similar trend for all cases. However, $T_{AMS\,|\,CD \geq cd}$ of the stations 2316 and 2325 tend to show higher return periods at the higher values of the AMS. These results indicate that this extreme drought events are less likely to happen in these regions. This may be due to the fact that these stations are located in high altitude areas in which snowmelt process can have impacts on streamflows, especially in spring. On the other hand, the conditional return periods are smaller for the station 2315, particularly at higher percentiles of AMS and CD values and meaning that extreme drought events are more likely to happen. These derived conditional return periods of the AMS and CD are crucial important to evaluate the risk which might be occurred when any water supply system cannot provide enough water under critical drought conditions.

Figure 6

Figure 7

4.4. Kendall Return Period

In the present work, supercritical risk of drought characteristics is also evaluated. Another definition of the bivariate return period introduced by Salvadori and De Michele (2004) and called as Kendall return period by Salvadori et al. (2011) are used to achieve this aim. Salvadori and De Michele (2010) reported that utilizing the standard definition of return period can bring about underestimates of the correct value, and they proposed the utilization of Kendall's return period (Mirabbasi et al. 2012). The Kendall return period can be defined as the average time between the occurrences of two supercritical drought events (Vandenberghe et al., 2011). The Kendall return period (also called the secondary return period) for drought duration and severity is defined as follows;

$$T^{*}_{AMS,CD} = \frac{1}{1-K_C(t)} = \frac{1}{1-P\big(C(F_{AMS,CD}(ams,cd))\leq t\big)} \tag{20}$$

In this equation, t denotes the critical probability level, K_c indicates Kendall's distribution function that can be easily computed for copulas of Archimedean family as follows;

$$K_c(t) = t - \frac{\varphi(t)}{\varphi'(t)} \tag{21}$$

Where denotes the right derivative of the generating function which is associated with the copula type. For instance, the generating function of the Gumbel-Hougaard copula can be defined as;

(22)

$$K_c(t) = \frac{t \cdot \theta - t \cdot \ln(t)}{\theta}$$

If the analytical form of K_c is not existed for any selected copula, it can be calculated using an algorithm proposed in Salvadori et al. (2011). Using equations 20-22, the Kendall's periods of the AMS and CD series were estimated for various critical probability levels (t). These levels were computed as follows (Vandenberghe et al. 2011);

$$t = 1 - \frac{1}{\check{T}_{AMS,CD}} \tag{23}$$

Table 7 represents the comparison of the univariate and bivariate return periods of the AMS and CD series. For instance, at the station 2315, which is located in the entrance of Muratlı Dam and HEPP (see Figure 1), if the engineers only consider single drought characteristics, then the univariate return period is assumed to be 100 years meaning that the AMS and CD are greater than the 4219 m^3/s days and 123 days, respectively. However, the bivariate return periods for those variables are 82, 129 and 115 years for case of $\hat{T}_{AMS,CD}$, $\check{T}_{AMS,CD}$ and $T^*_{AMS,CD}$, respectively. Furthermore, as can be seen from Table 7, the Kendall's return periods are always larger than the bivariate return period $\check{T}_{AMS,CD}$ and are always smaller than $\hat{T}_{AMS,CS}$. It should be noted that the Kendall's (secondary) return period ($T^*_{AMS,CD}$) is totally different from the primary return periods ($\hat{T}_{AMS,CD}$, $\check{T}_{AMS,CD}$) as they describe different situations. Therefore, it is not possible to say which performs consistently better than the others (Serinaldi, 2015). Importance of these return periods only changes based on which one better describes the assessment and management requirements of drought risks in the studied region. The relative differences between the univariate return period and $\widehat{T}_{AMS,CD}$ are also provided in Table 7. A lower relative difference indicates a higher drought risk. It is clearly observed from the table that the differences of stations 2322, 2316, 2325 and 2323 is relatively higher than those of the stations 2305, 2315 and 2320. Station 2322 has the highest difference while the station 2315 has the lowest relative difference indicating the highest drought risk. The main reasons of these differences may be the spatial-temporal variations in precipitation in this basin and constructed dams (e.g., regulations of water reservoirs).

Table 7

5. Summary and Conclusions

This study represents the first research to model joint distribution functions of annual maximum drought severity and corresponding duration via bivariate copulas. The Çoruh Basin which is one of the most important water resources of Turkey was selected for the study. Drought characteristics were computed from daily streamflow data of the seven gauging stations using threshold level method. Various marginal distributions were evaluated to fit annual maximum severity and corresponding duration series for all stations. Archimedean family of copulas including Ali-Mikhail-Haq, Clayton, Frank and Gumbel-Hougaard were considered for modelling joint distribution of correlated drought characteristics. The following conclusions can be drawn from this study:

1- Determination of marginal distributions for drought characteristics is essential and crucial step for multivariate modelling. Therefore, eleven widely used univariate distributions were fitted and compared. Akaike Information Criterion (AIC) was employed to determine the most suitable distribution among the candidate distributions. The AIC test results indicated that the annual maximum severity series were best fitted either by the Exponential and Weibull distributions while the corresponding duration series were well fitted either by the Weibull and Logistic distributions.

2- After evaluating goodness-of-fit tests, upper tail and graphical assessments, the Gumbel-Hougaard copulas were selected as the most suitable copula type for modelling dependence structure of the drought characteristics.

3- The conditional and joint return periods of the drought characteristics were derived using the developed Gumbel-Hougaard copulas for each station. As an alternative joint return period, Kendall's return period, was also assessed and the univariate and joint return periods of the drought characteristics were compared. As a result, the bivariate return periods of the annual maximum severity and corresponding duration characteristics can provide more useful information for reliable drought risk assessments in the basin. Through the paper, we also specified that the bivariate Kendall's return periods and standard bivariate return periods cannot be interchanged as their applicability depends on the type of drought risk considered.

4- Comparison of the univariate and bivariate return periods showed that the stations 2305, 2315 and 2320 have a higher drought risk than the stations 2322, 2316, 2325 and 2323 in Çoruh Basin. Station 2322 has the lowest drought risk while the station 2315 has the highest drought risk. In order to decrease drought effects in this basin, accurate water resources management (e.g., hydrological regulations of water reservoirs) of the Çoruh Basin is necessary.

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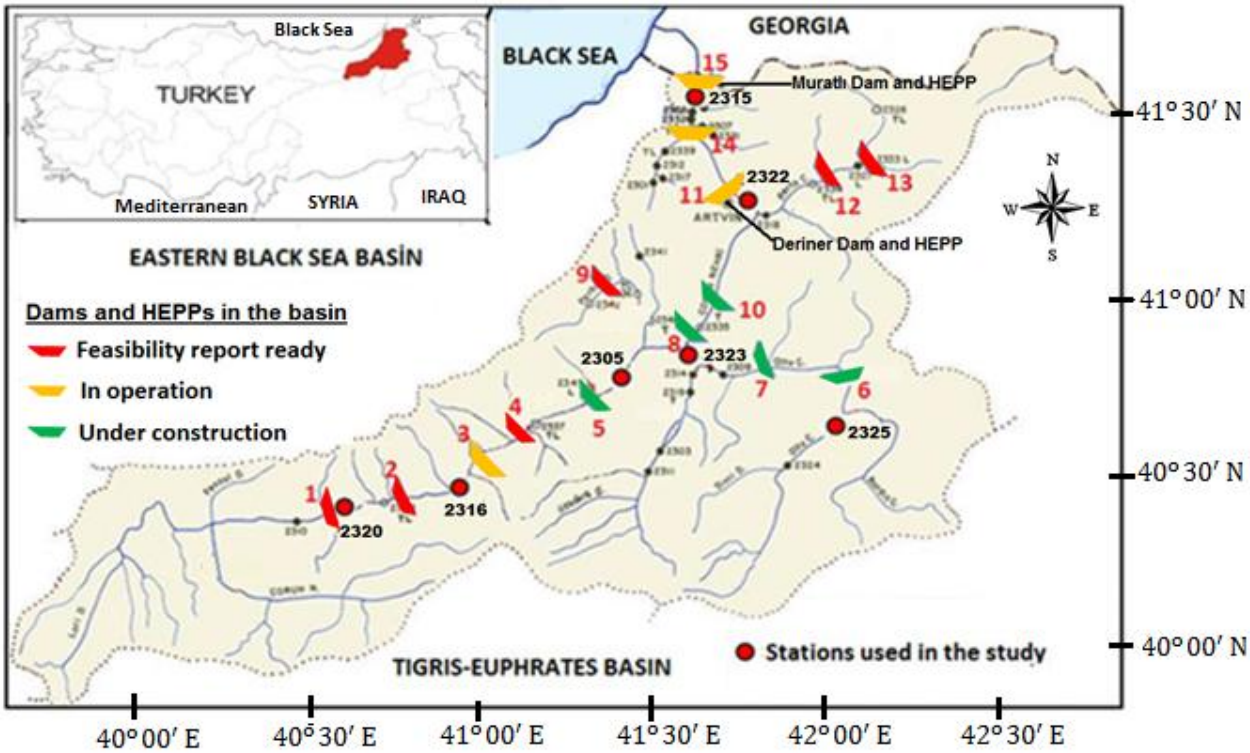


Figure 1. Location map of the considered stations and Dams and HEPPs in the Çoruh Basin

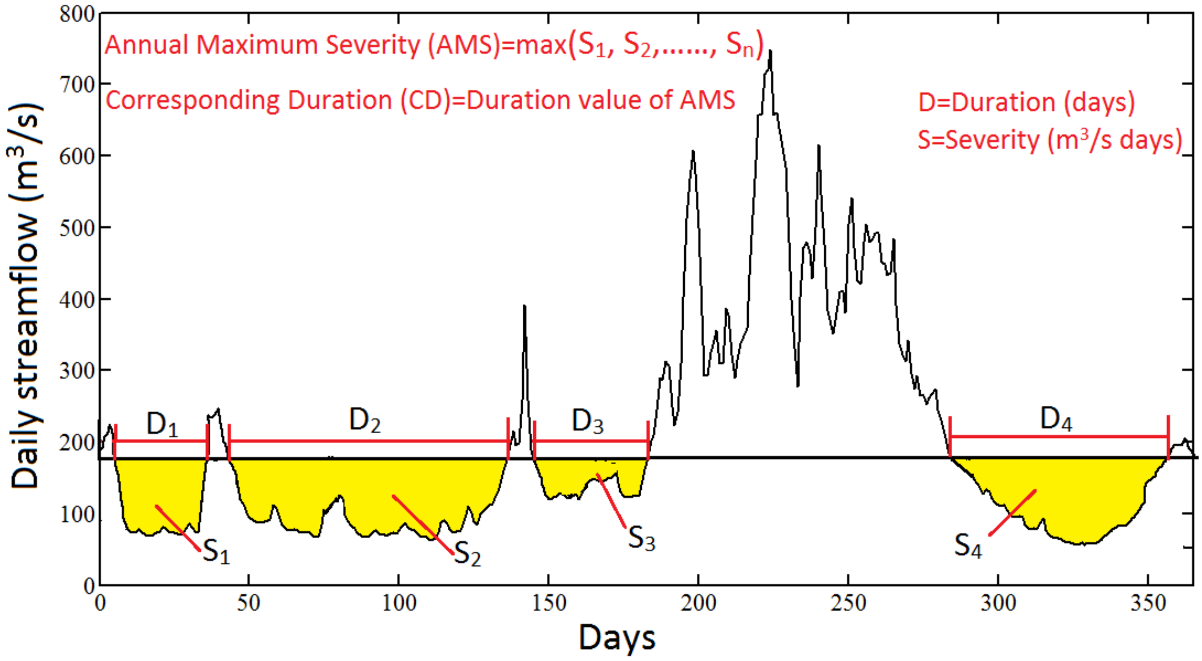


Figure 2. Definition of the drought characteristics considered in the study

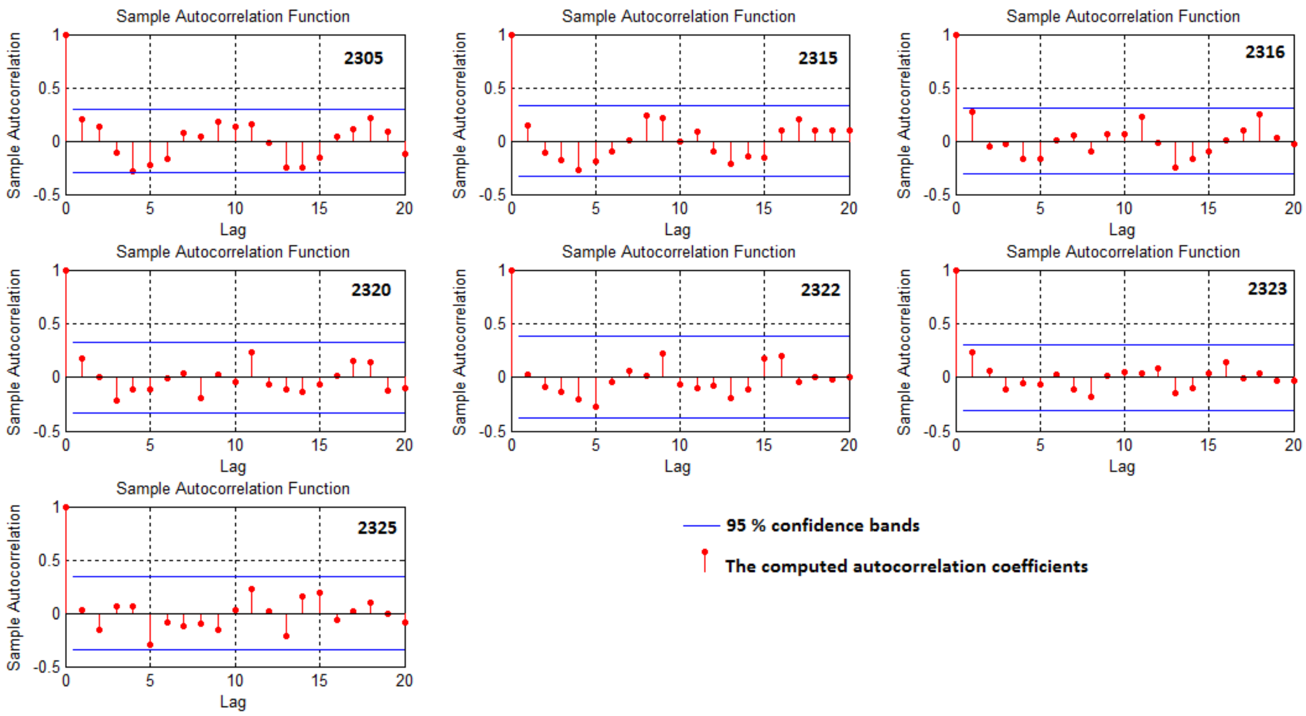


Figure 3a. Autocorrelation plot of the AMS series for all stations

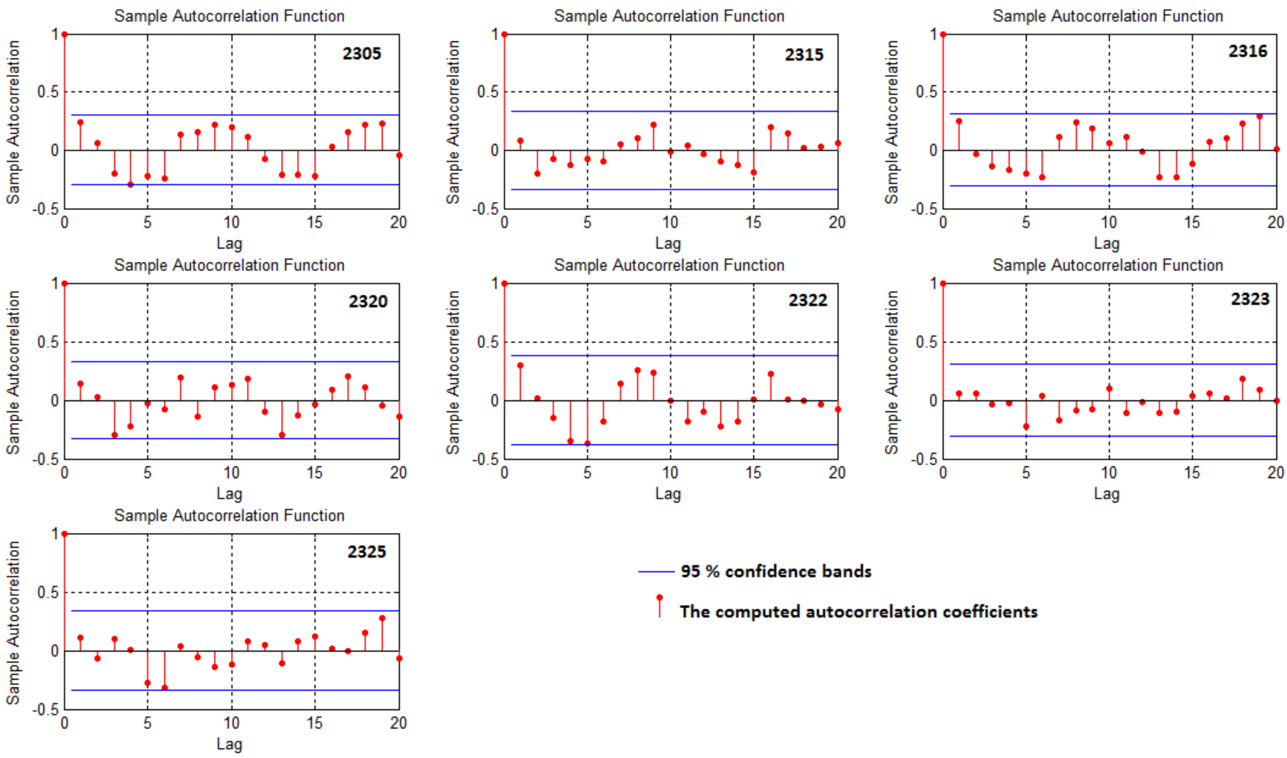


Figure 3b. Autocorrelation plot of the CD series for all stations

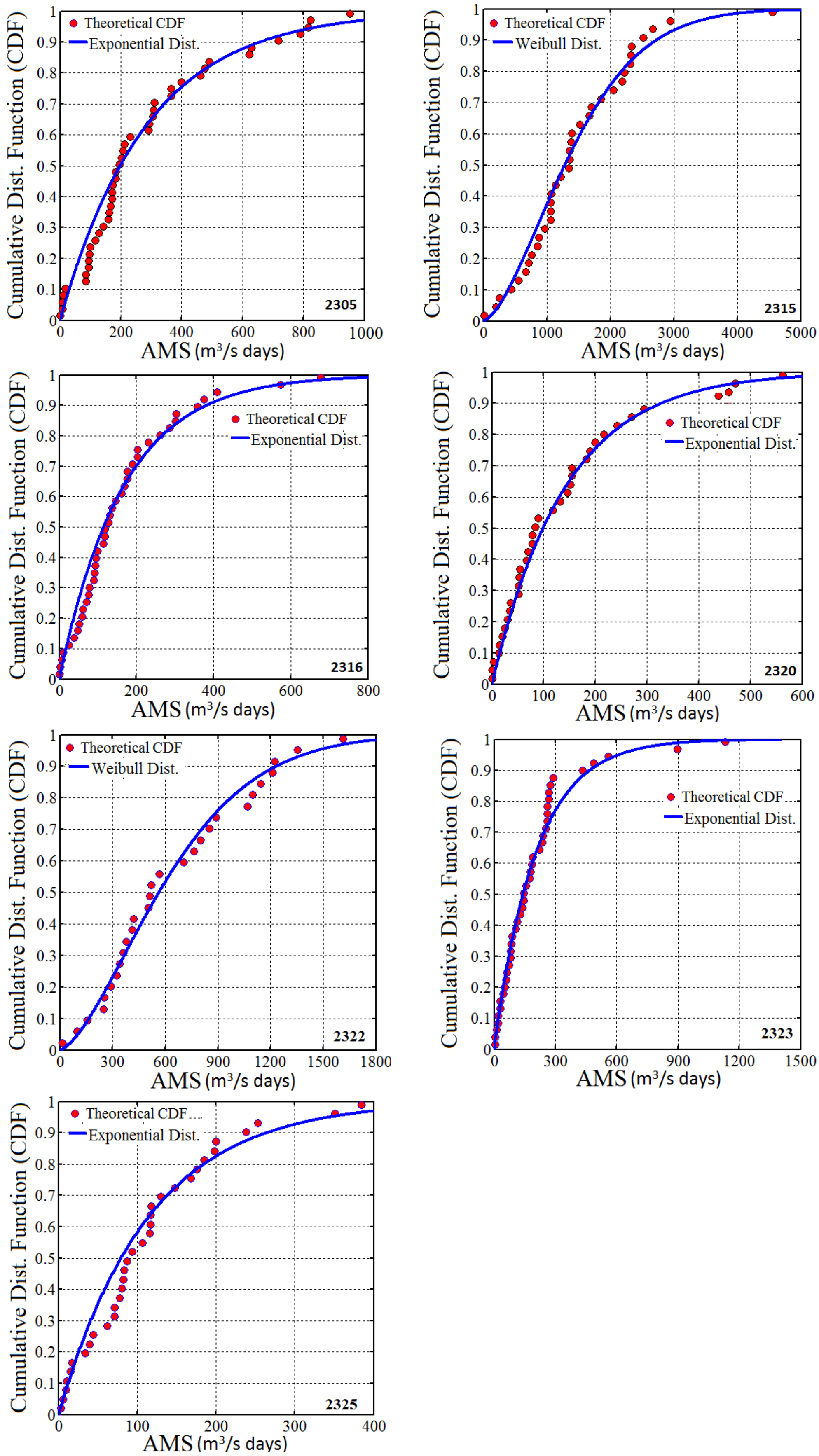


Figure 4a. Comparison between empirical and theoretical CDFs for the AMS series

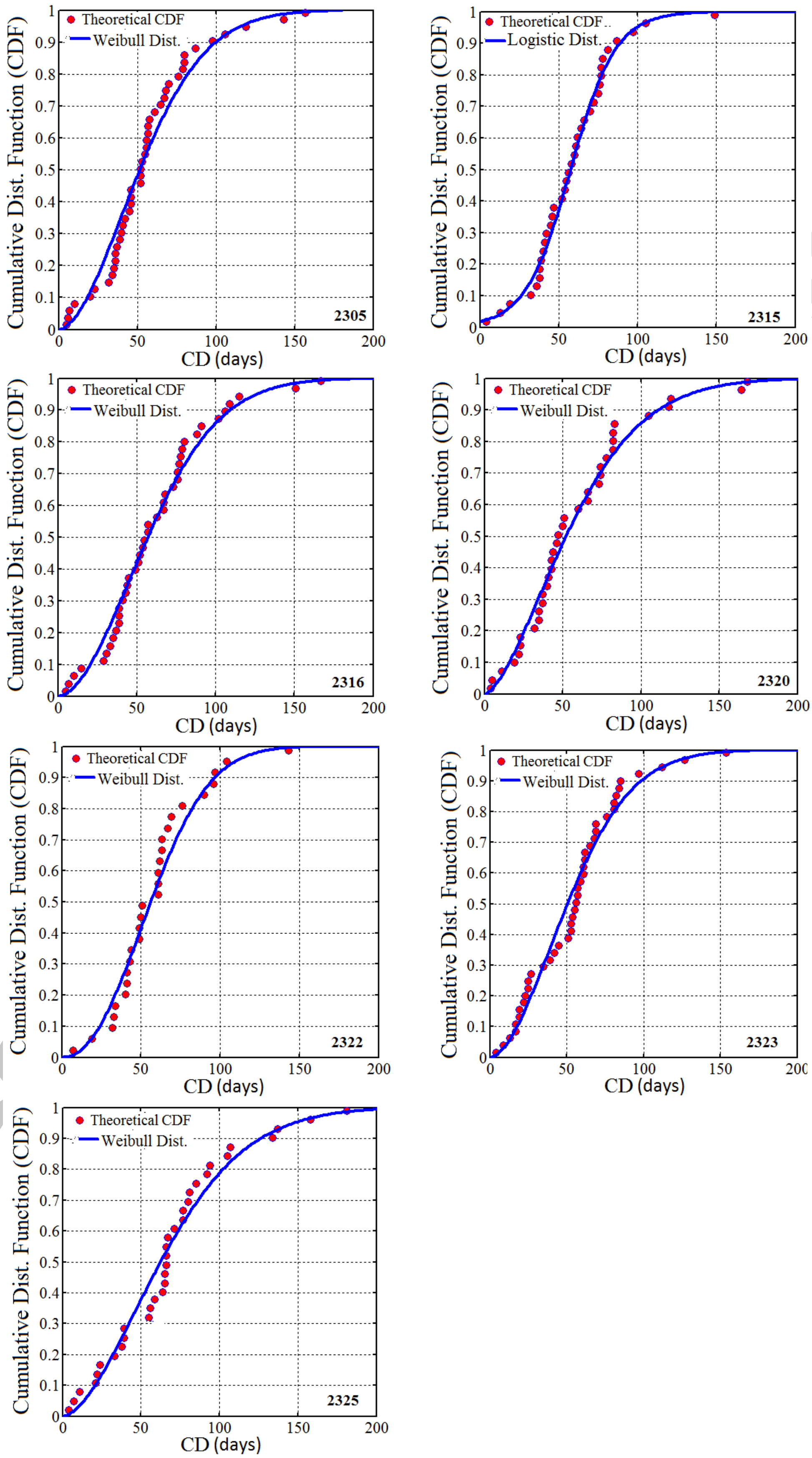


Figure 4b. Comparison between empirical and theoretical CDFs for the CD series

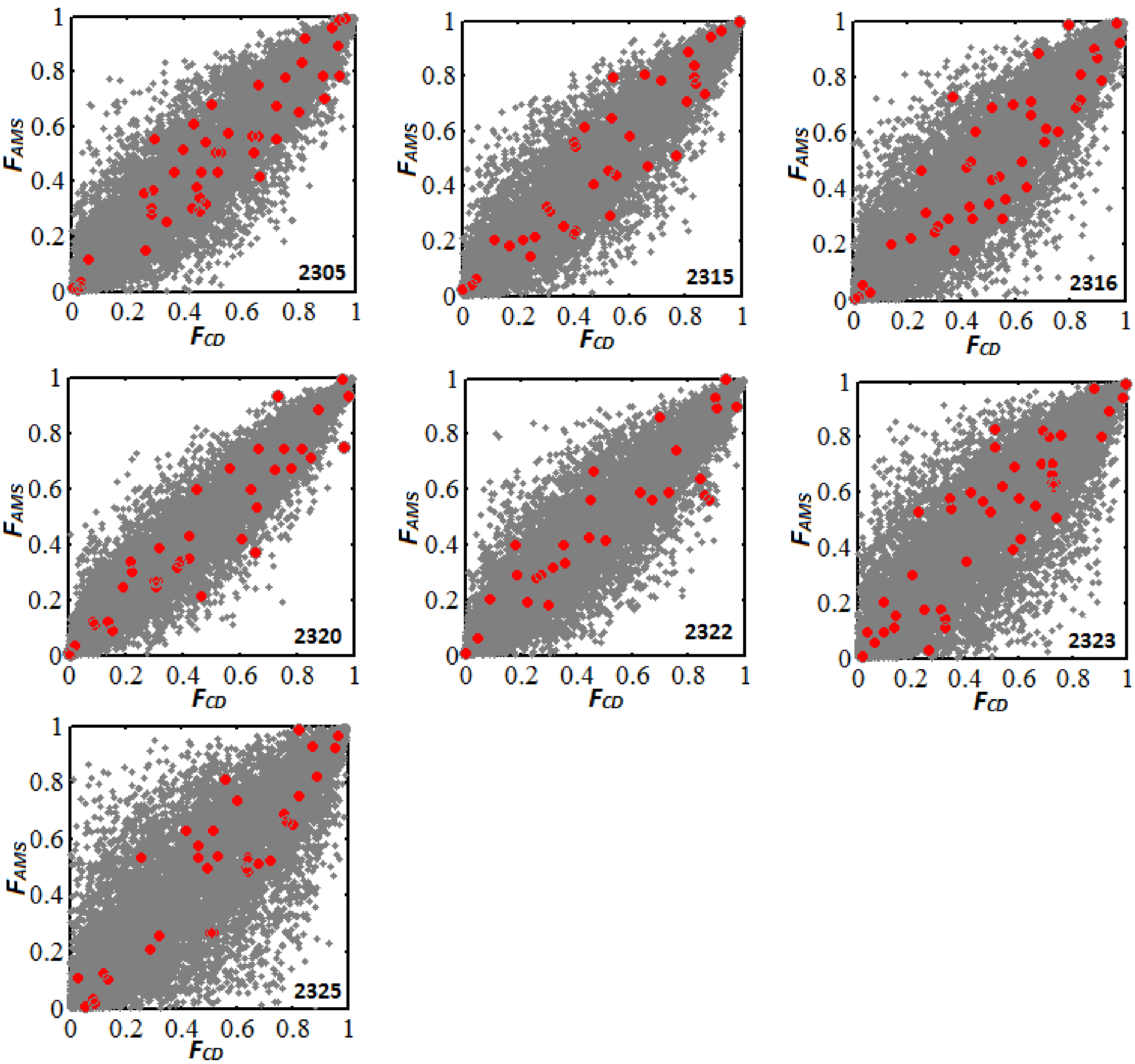
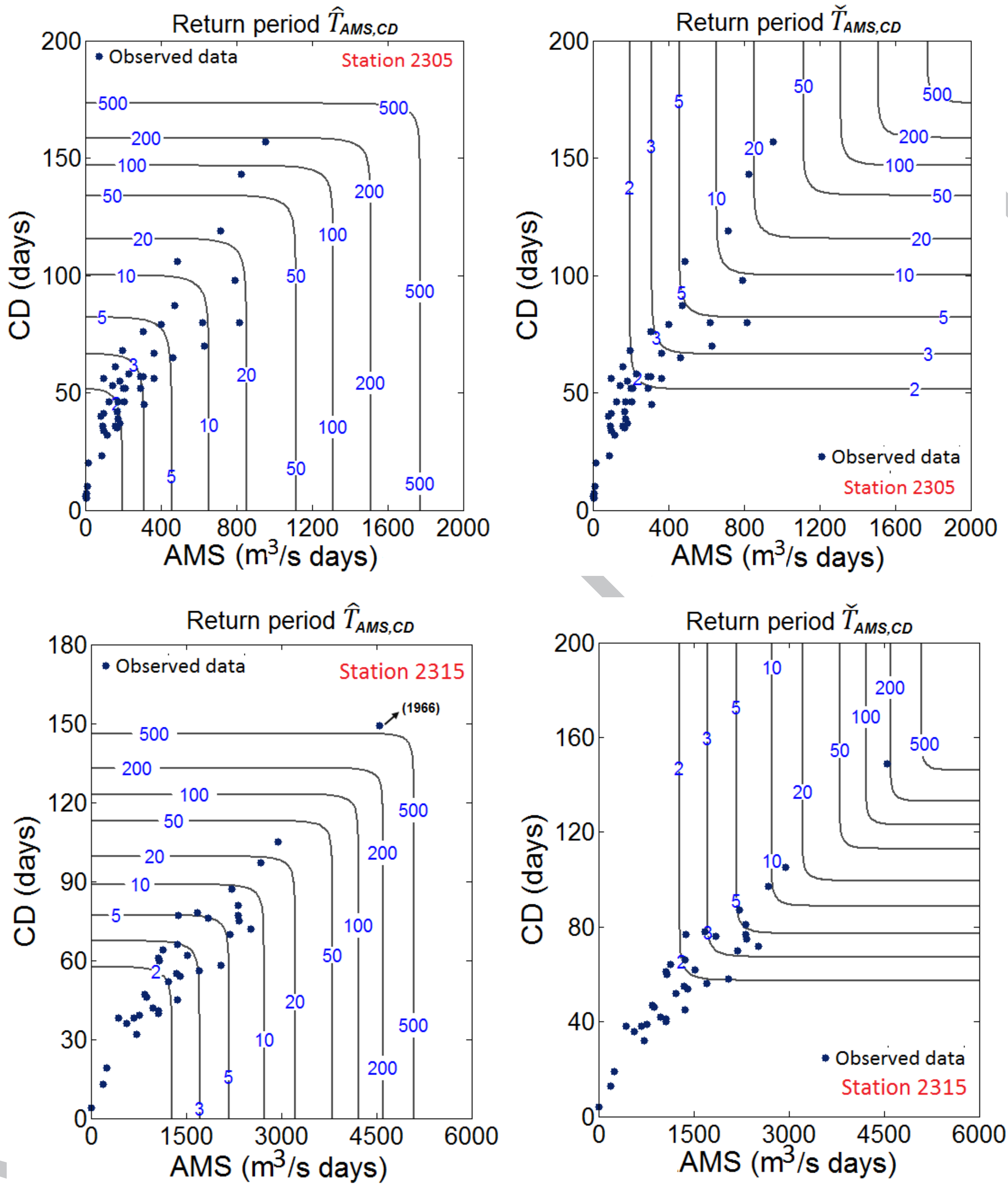
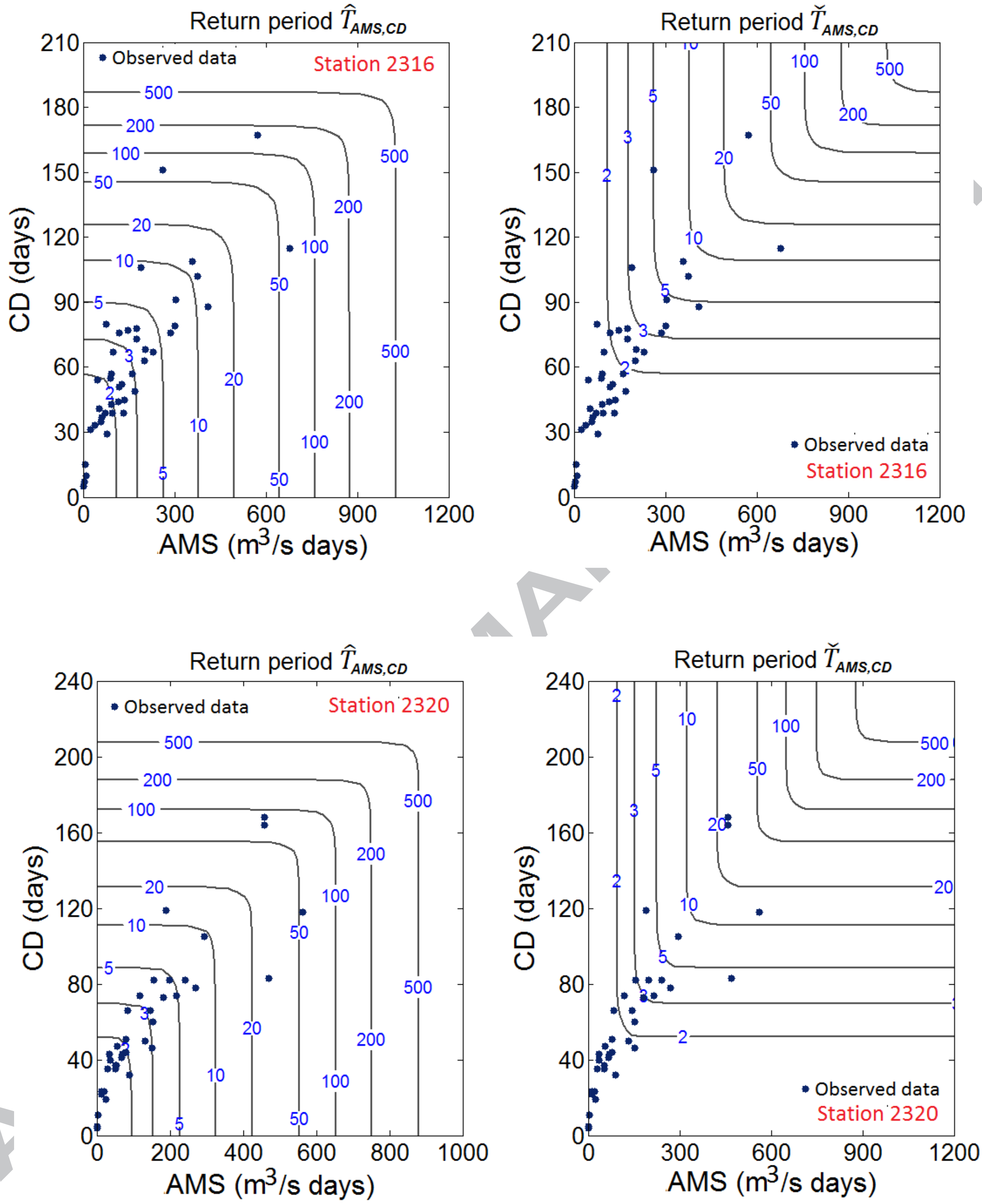
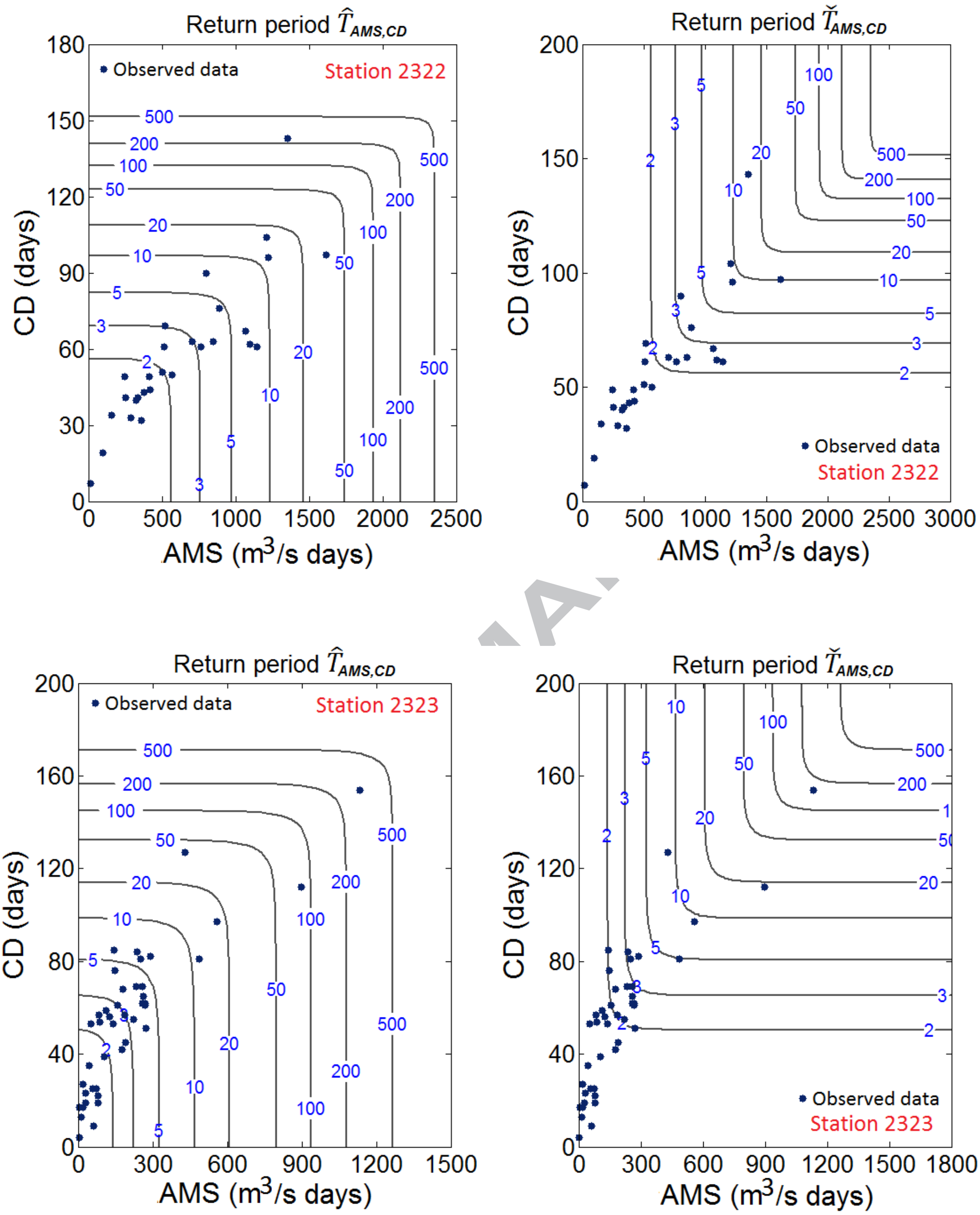


Figure 5. Scatter plots of historical values versus generated samples


Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2305
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2305
CD (days)
AMS (m^3/s days)
Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2315
(1966)
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2315
CD (days)
AMS (m^3/s days)


Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2316
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2316
CD (days)
AMS (m^3/s days)
Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2320
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2320
CD (days)
AMS (m^3/s days)


Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2322
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2322
CD (days)
AMS (m^3/s days)
Return period $\hat{T}_{AMS,CD}$
Observed data
Station 2323
CD (days)
AMS (m^3/s days)
Return period $\check{T}_{AMS,CD}$
Observed data
Station 2323
CD (days)
AMS (m^3/s days)

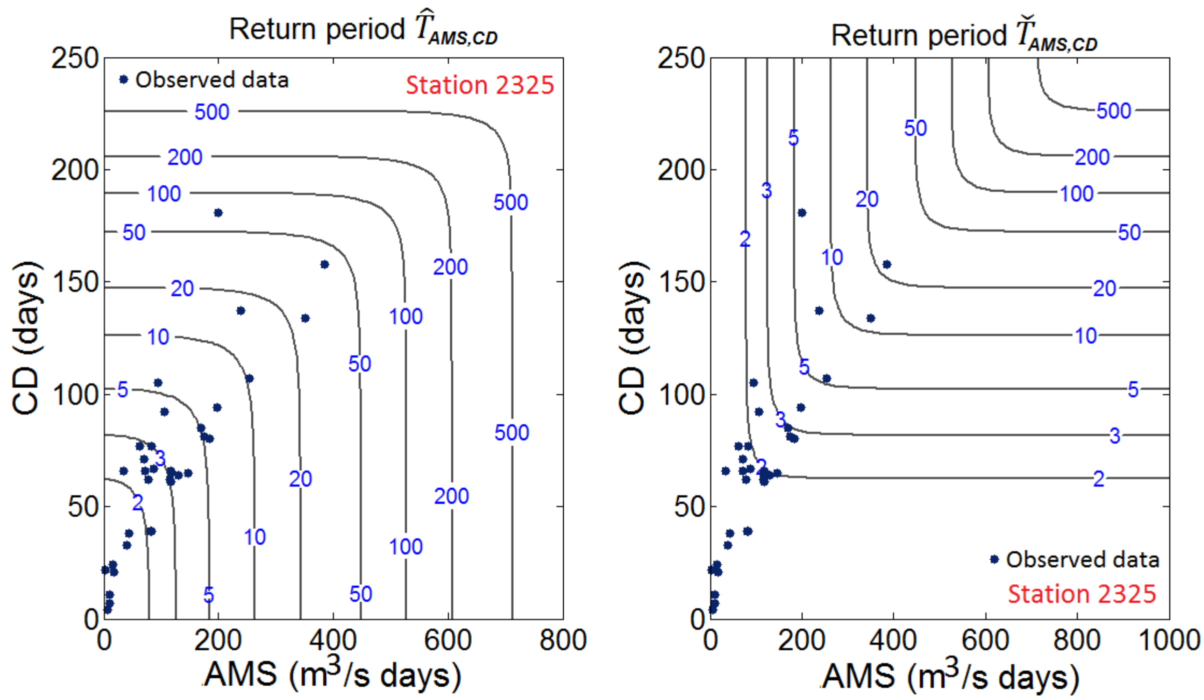
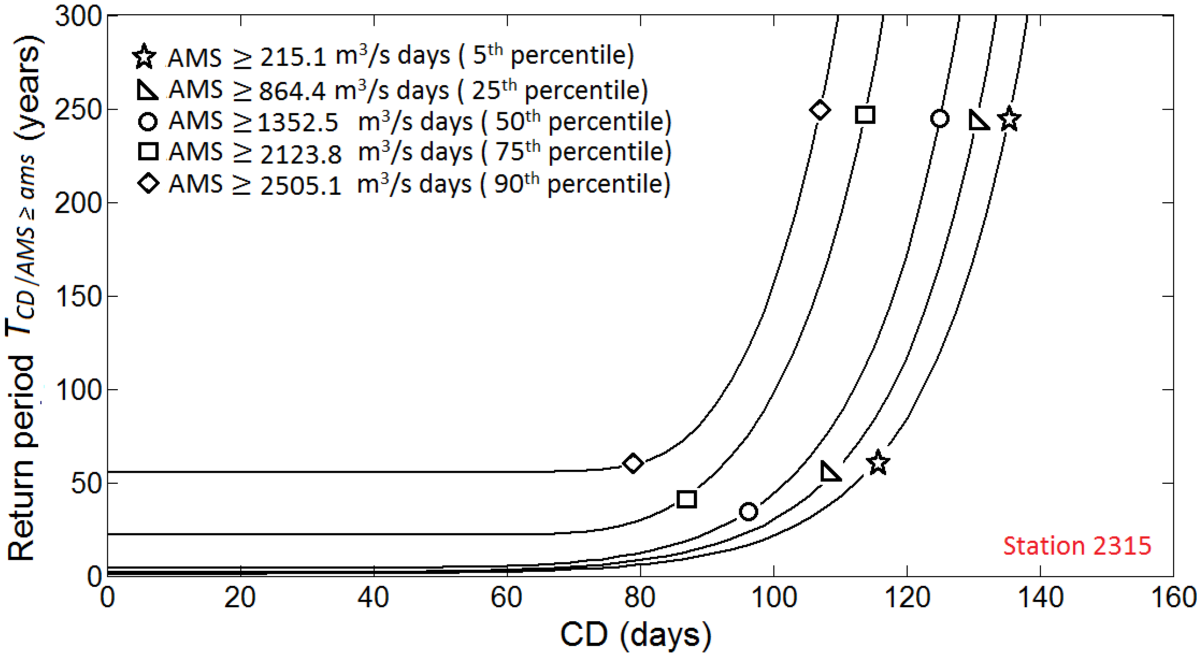
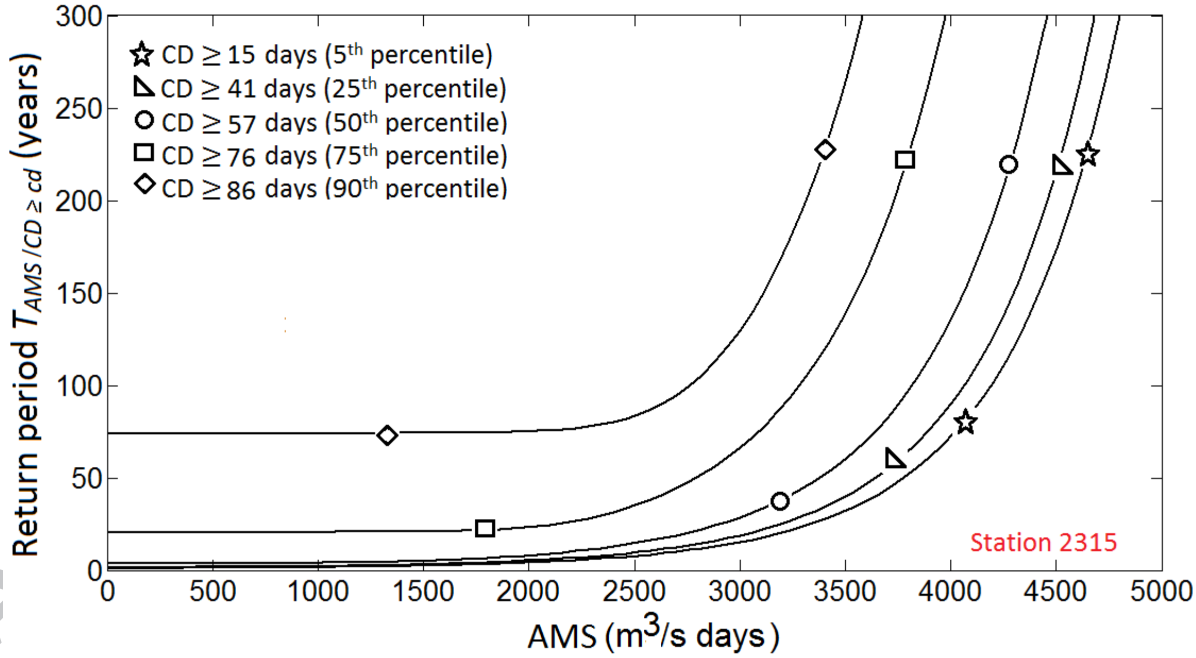
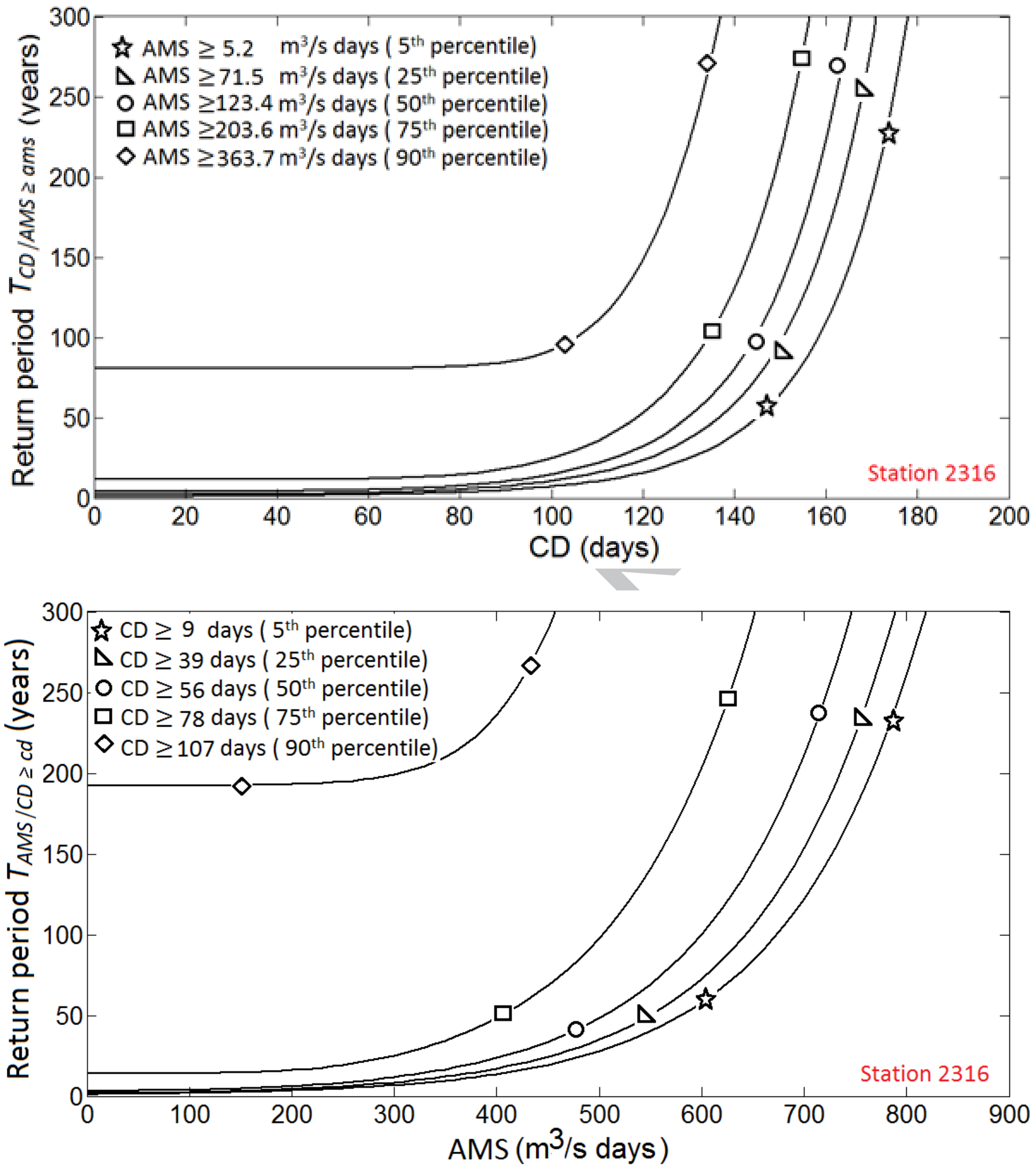
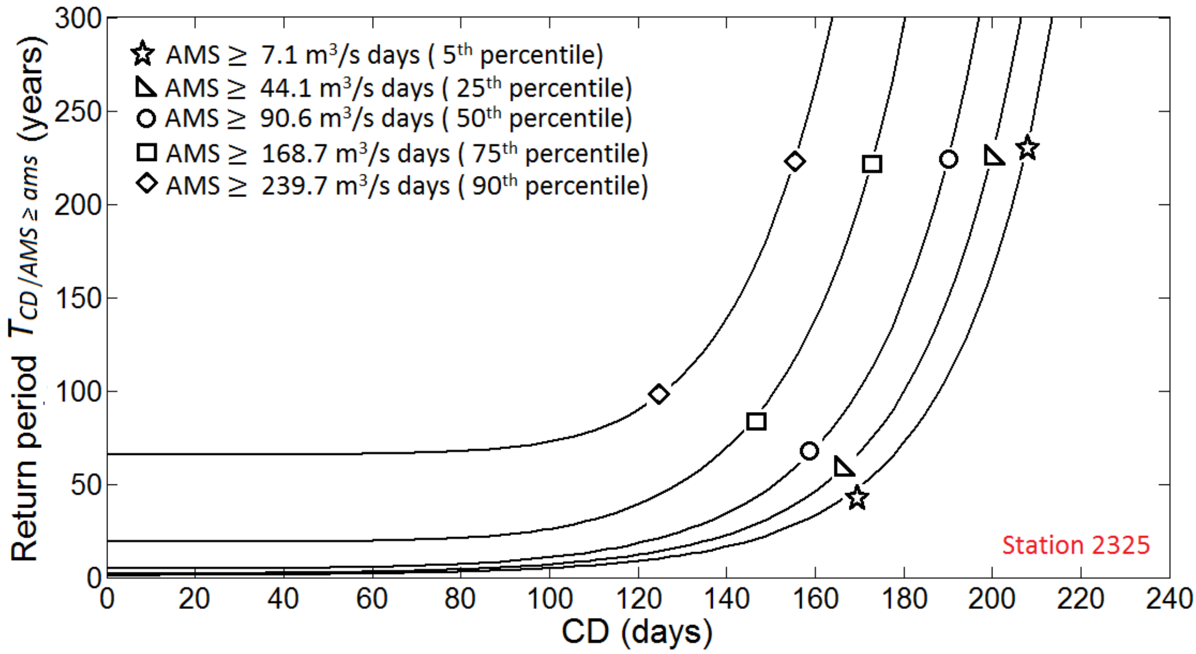


Figure 6. The joint return periods ($\hat{T}_{AMS,CD}$ and $\check{T}_{AMS,CD}$) of the AMS and CD series


AMS ≥ 215.1 m³/s days (5th percentile)
AMS ≥ 864.4 m³/s days (25th percentile)
AMS ≥ 1352.5 m³/s days (50th percentile)
AMS ≥ 2123.8 m³/s days (75th percentile)
AMS ≥ 2505.1 m³/s days (90th percentile)
Station 2315
Return period $T_{CD/AMS \geq ams}$ (years)
CD (days)


CD ≥ 15 days (5th percentile)
CD ≥ 41 days (25th percentile)
CD ≥ 57 days (50th percentile)
CD ≥ 76 days (75th percentile)
CD ≥ 86 days (90th percentile)
Station 2315
Return period $T_{AMS/CD \geq cd}$ (years)
AMS (m³/s days)


AMS ≥ 5.2 m³/s days (5th percentile)
AMS ≥71.5 m³/s days (25th percentile)
AMS ≥123.4 m³/s days (50th percentile)
AMS ≥203.6 m³/s days (75th percentile)
AMS ≥363.7 m³/s days (90th percentile)
Return period $T_{CD/AMS \geq ams}$ (years)
CD (days)
Station 2316
CD ≥ 9 days (5th percentile)
CD ≥ 39 days (25th percentile)
CD ≥ 56 days (50th percentile)
CD ≥ 78 days (75th percentile)
CD ≥107 days (90th percentile)
Return period $T_{AMS/CD \geq cd}$ (years)
AMS (m³/s days)
Station 2316



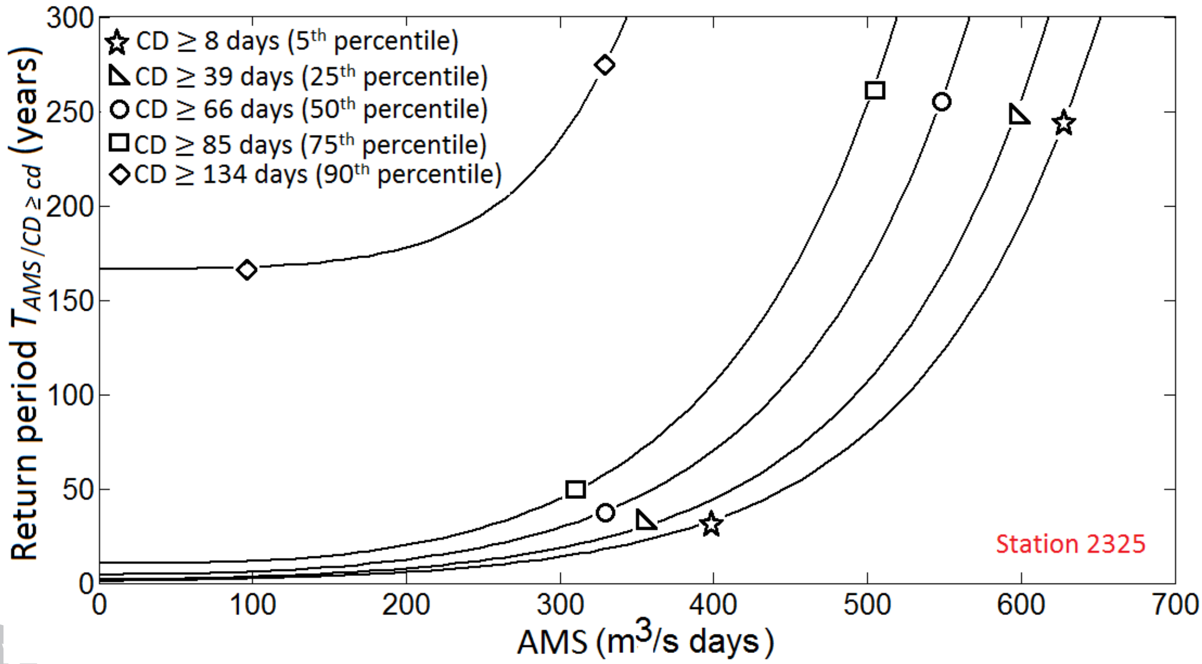


Figure 7. The conditional return periods ($T_{CD\,|\,AMS \geq ams}$ and $T_{AMS\,|\,CD \geq cd}$) of the AMS and CD series for the selected stations

Table 1. Summary description of the bivariate Archimedean Copulas evaluated in the study

Copula type	Bivariate Copula C(u,v)	Parameter range (θ)	Generating Function ($\varphi(t)$)	Relation of Kendal's τ and θ
Ali-Mikhail-Haq	$\frac{u \cdot v}{1-\theta(1-u)(1-v)}$	$-1 \leq \theta \leq 1$	$ln\left(\frac{1-\theta(1-t)}{t}\right)$	$\tau(\theta) = \left(1-\frac{2}{3\theta}\right)-\frac{2}{3}\left(1-\frac{1}{\theta}\right)^2 ln(1-\theta)$
Clayton	$\left[u^{-\theta}+v^{-\theta}-1\right]^{-\frac{1}{\theta}}$	$\theta > 0$	$\frac{1}{\theta}\left(t^{-\theta}-1\right)$	$\tau(\theta) = \frac{\theta}{\theta+2}$
Frank	$-\frac{1}{\theta} ln\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right]$	$\theta \neq 0$	$-ln\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\tau(\theta) = 1+4\left(\frac{D_1(-ln\theta)-1}{ln\theta}\right)$ Where $D_k(x)$ is the Debye function; for any positive integer k, $D_k(x) = {}^{k}/_{x^2} \int_0^x {}^{t^k}/_{(e^t-1)}\, dt$ (Wang et al. 2009)
Gumbel-Hougaard	$exp\left\{-\left[(-ln(u))^{\theta}+(-ln(v))^{\theta}\right]^{\frac{1}{\theta}}\right\}$	$\theta \geq 1$	$(-ln(t))^{\theta}$	$\tau(\theta) = \frac{\theta-1}{\theta}$

Table 2. Mann-Kendall test results for the AMS and CD series of the selected stations

Basic properties of the streamflow and drought data									Mann-Kendall test results			
Station Number	Drainage area (km^2)	Elevation (m)	Observ. Period	$\bar{X}$ (m^3/s)	SD (m^3/s)	Parameter	$\bar{X}^*$	SD^*	Calculated Z value	Critical Z Value, α=0.05	Ho Hypoth.	Trend
2305	7272	654	1963-2007 (45 years)	70.5	80.1	AMS (m^3/s days)	285.2	223.0	0.07	±1.96	Accept	No
						CD (days)	56.2	32.2	-0.85	±1.96	Accept	No
2315	20,127	57	1965-2000 (36 years)	209.1	207.6	AMS (m^3/s days)	1464.5	907.1	-0.50	±1.96	Accept	No
						CD (days)	58.7	27.1	-0.72	±1.96	Accept	No
2316	5505.2	1170	1966-2007 (42 years)	39.4	48.5	AMS (m^3/s days)	165.7	147.1	1.06	±1.96	Accept	No
						CD (days)	61.9	34.8	0.20	±1.96	Accept	No
2320	4759.2	1365	1971-2007 (37 years)	29.4	36.2	AMS (m^3/s days)	142.2	145.2	-0.22	±1.96	Accept	No
						CD (days)	58.9	38.7	0.13	±1.96	Accept	No
2322	18,753.3	201	1972-1999 (28 years)	160.1	172.9	AMS (m^3/s days)	648.2	420.1	-0.10	±1.96	Accept	No
						CD (days)	58.8	28.2	-0.53	±1.96	Accept	No
2323	7069.8	580	1965-2007 (43 years)	33.9	39.9	AMS (m^3/s days)	203.8	222.9	0.77	±1.96	Accept	No
						CD (days)	55.2	31.9	-0.32	±1.96	Accept	No
2325	1762	1129	1974-2007 (34 years)	7.2	10.2	AMS (m^3/s days)	114.6	93.0	-0.81	±1.96	Accept	No
						CD (days)	69.4	41.2	-0.73	±1.96	Accept	No

Table 3. The best fitted univariate distributions for the AMS and CD series of each station

Station No	Drought variables	Distribution	Parameters	AIC	PDF
2305	AMS	Exponential	α= 285.23	600.79	Exponential; $f(x) = \frac{1}{\alpha} e^{\left(\frac{-x}{\alpha}\right)}, \quad x > 0$ Weibull; $f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}}, \quad 0 < x < \infty$ Logistic; $f(x) = \frac{1}{\alpha} e^{\left(\frac{x-\beta}{\alpha}\right)}\left[1 + e^{\left(\frac{x-\beta}{\alpha}\right)}\right]^{-2}, \quad -\infty < x < \infty$
	CD	Weibull	α= 63.06 β= 1.81	436.82	
2315	AMS	Weibull	α= 1617.08 β= 1.59	590.87	
	CD	Logistic	α= 14.27 β= 57.44	340.09	
2316	AMS	Exponential	α= 165.71	515.26	
	CD	Weibull	α= 69.52 β= 1.85	415.08	
2320	AMS	Exponential	α= 142.16	442.81	
	CD	Weibull	α= 65.72 β= 1.59	369.18	
2322	AMS	Weibull	α= 716.21 β= 1.54	416.03	
	CD	Weibull	α= 66.28 β= 2.21	267.37	
2323	AMS	Exponential	α= 203.82	545.28	
	CD	Weibull	α= 61.94 β= 1.80	416.82	
2325	AMS	Exponential	α= 114.60	392.42	
	CD	Weibull	α= 77.44 β= 1.71	348.53	

Table 4. Correlation coefficients for drought characteristics

Station No	Pearson's (ρ)	Kendall's (τ)
2305	0.897 (6.9×10^{-17})	0.743 (8.3×10^{-13})
2315	0.941 (1.6×10^{-17})	0.760 (8.1×10^{-11})
2316	0.820 (3.2×10^{-11})	0.707 (5.1×10^{-11})
2320	0.874 (1.7×10^{-12})	0.806 (3.3×10^{-12})
2322	0.862 (3.7×10^{-9})	0.757 (2.2×10^{-8})
2323	0.844 (1.1×10^{-12})	0.670 (3.1×10^{-10})
2325	0.843 (4.1×10^{-10})	0.639 (1.4×10^{-7})

Bracketed values indicate the computed p values

Table 5. Results of goodness of fit tests for the candidate copulas and parameter values of the best fitted copulas

Station No	Clayton			Frank			Gumbel-Hougaard			Parameter of best fitted Copula (θ)
	D_{IAD}	S_n	p value	D_{IAD}	S_n	p value	D_{IAD}	S_n	p value	
2305	0.225	0.04034	0.0095	0.148	0.02812	0.1082	0.108	0.0194	0.4825	3.893
2315	0.089	0.02782	0.1284	0.082	0.02481	0.2988	0.071	0.0210	0.4918	4.166
2316	0.145	0.03585	0.0322	0.093	0.02701	0.2071	0.063	0.0220	0.3941	3.418
2320	0.165	0.03008	0.0481	0.090	0.01989	0.5096	0.073	0.0191	0.5183	5.148
2322	0.136	0.03636	0.0627	0.097	0.03391	0.2298	0.082	0.0327	0.1921	4.121
2323	0.157	0.02932	0.1348	0.143	0.02620	0.2682	0.127	0.0253	0.2761	3.030
2325	0.244	0.05111	0.0103	0.235	0.04408	0.0349	0.197	0.0357	0.0916	2.767

Table 6. Parametric and Non-parametric coefficients of upper tail dependence for all stations

Station	λ_U^{CFG}	λ_U
2305	0.761	0.805
2315	0.772	0.819
2316	0.728	0.775
2320	0.770	0.856
2322	0.742	0.817
2323	0.735	0.742
2325	0.722	0.715

Table 7. Comparison of univariate and bivariate return periods for drought characteristic

Station No	Return Period (RP) (years)	AMS (m^3/s days)	CD (days)	$\breve{T}_{AMS,CD}$ (years)	$\widehat{T}_{AMS,CD}$ (years)	$T^*_{AMS,CD}$ (years)	Relative difference between RP and $\widehat{T}_{AMS,CD}$ (%)

2305	50	1116	134	41	65	58	23
	100	1314	147	81	130	116	23
	200	1511	159	162	261	232	23
2315	50	3808	113	41	64	58	22
	100	4219	123	82	129	115	22
	200	4607	133	163	258	231	22
2316	50	648	145	39	68	60	26
	100	763	159	79	137	120	27
	200	878	171	157	275	241	27
2320	50	556	155	41	65	58	23
	100	655	172	81	130	116	23
	200	753	188	162	261	232	23
2322	50	1741	123	39	70	61	29
	100	1936	132	78	140	122	29
	200	2121	141	156	280	244	29
2323	50	797	132	40	67	59	25
	100	939	145	80	134	119	25
	200	1080	157	159	269	237	26
2325	50	448	172	40	68	59	26
	100	528	190	79	135	119	26
	200	607	206	158	271	239	26

Highlights

- The Exponential, Weibull and Logistic distributions were the best fitted marginal distributions for AMS and CD series.
- Gumbel-Hougaard copulas were selected as the most suitable bivariate distributions for joint modelling of the AMS and CD series at stations.
- The derived GH copulas were used to derive conditional and joint return periods of various AMS and CD pairs.
- According to the calculated relative differences, the highest drought risk was obtained for the station 2315 while the lowest drought risk was appeared for the station 2322.