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An enhanced trend surface analysis equation for regional–residual separation of gravity data

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ABSTRACT

Trend surface analysis is a geological term for a mathematical technique which separates a given map set into a regional component and a local component. This work has extended the steps for the derivation of the constants in the trend surface analysis equation from the popularly known matrix and simultaneous form to a more simplified and easily achievable format. To achieve this, matrix inversion was applied to the existing equations and the outcome was tested for suitability using a large volume of gravity data set acquired from the Anambra Basin, south-eastern Nigeria. Tabulation of the field data set was done using the Microsoft Excel spread sheet, while gravity maps were generated from the data set using Oasis Montaj software. A comparison of the residual gravity map produced using the new equations with its software derived counterpart has shown that the former has a higher enhancing capacity than the latter. This equation has shown strong suitability for application in the separation of gravity data sets into their regional and residual components.

1. Introduction

Simply put, trend surface analysis is a geological term for a mathematical technique which separates a given map set into two components namely — a regional component and a local component (Davis, 2014). Grant (1961) defined it as that part of data that varies smoothly. Invariably, it is a function that runs in a predictive pattern. It is associated with large scale systematic changes which extend from one map edge to the other (Krumbein and Graybill, 1965). It tries to decompose every observation made on a spatial plane into their regional and local component effects respectively (Unwin, 1978) by introducing a line of best fit on the entire data set using the regression method. The outcome of such analysis becomes the Regional effect, while individual point variations from the regional effect are known as the assumed error or residuals or local component. The problem of clustering of sampled points and spatial auto correlation of residual values was earlier identified with trend surface analysis of which a solution has been proffered (Norcliffe, 1969). The Residuals occur in a non-systematic pattern, superimposed on the regional pattern and appear to be spatially random (Krumbein and Graybill, 1965). Trend surface analysis has found its application in many branches of study ranging from agriculture, to geography to ecology (Tobler, 1966; Chorley and Haggett, 1965) to geology (Krumbein, 1959; Grant, 1961; Davis, 2014) and even in industries (Davies, 1954; Hill and Hunter, 1968). The application of trend surface analysis in geology tries to solve two main forms of geologic problems, an aspect of which is the fitting of structural data into its regional component and local component, as it is often the case in geophysics. The second form of the problem is common in petrography and geochemistry (Davis, 2014). This method was recently applied in the analysis of potential field data (Likkason, 1993; Olowofela et al., 2006; Okiwelu et al., 2010; Opara, 2011). The principles and some advances in the application of trend surface analysis have been widely reported (Agterberg, 1984; Weisberg, 1985; Zimmerman et al., 1996). Previous researchers stopped the equation at the identity matrix (Unwin, 1978; Davis, 2014) and referred readers to computer programs for the analysis of large data sets, which would hardly be solved using simultaneous equations. This has generated a form of ambiguity and gap in knowledge, as young scholars in the geosciences find it very difficult to appreciate the approach as handled by the computer. The aim of this work is to derive an equation which is easily handled and carried out without programming for gravity field separation. To achieve this, the existing matrix form of the equation was further subjected to matrix inversion, with relevant assumptions made where necessary.

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2. Material and methods

Matrix inversion was applied to the existing matrix form of trend surface equations to generate new sets of equations. The new equations were then tested using a gravity data set. The gravity data set as used in this work was acquired by the Nigeria Geological Survey Agency (NGSA) between 2008 and 2011 in the Anambra Basin of south-eastern Nigeria and its environs. A total of 16,641 data points were acquired. Both ground and air surveys were employed to ensure high data density. The Microsoft Excel spread sheet was used in tabulating the entire data set while Oasis Montaj software produced by Geosoft Incorporated was applied in plotting the gravity data set in contour maps and colour spectrum bands.

3. Theory/calculation

The Bouguer gravity value is a combination of the regional gravity value within the study area and point residual anomalies within the study area (Unwin, 1978; Davis, 2014). Hence,

$$\text{Bouguer Gravity value} = \text{Regional gravity value} + \text{Residual gravity value}$$
$$\text{i.e.} \quad \Delta g_B = \Delta g_R + \Delta g_r \tag{1}$$

where:

Δg_B Bouguer gravity value
Δg_R Regional gravity value
Δg_r Residual gravity value

Let:

$$\Delta g_B = Y_{ij} \tag{2}$$

$$\Delta g_R = ax_i + by_j + c \tag{3}$$

$$\Delta g_r = e_{ij} \tag{4}$$

Then, Eq. (1) becomes

$$Y_{ij} = \left(ax_i + by_j + c\right) + e_{ij} \tag{5}$$

where:

Y_{ij} Bouguer gravity readings
x_i Measurement points in the x-direction
y_j Measurement points in the y-direction
e_{ij} Residual gravity readings.

a, b, and c are constants.

Hence, the residual is given as

$$e_{ij} = Y_{ij} - \left(ax_i + by_j + c\right) \tag{6}$$

4. Results

Let S = sum of the squares of the residuals, e_{ij}.
Hence,

$$S = \sum_{\substack{i=1\\j=1}}^{N} e_{ij}^2 \tag{7}$$

$$\Rightarrow S = \sum_{\substack{i=1\\j=1}}^{N} e_{ij}^2 = \sum_{\substack{i=1\\j=1}}^{N} \left[Y_{ij} - \left(ax_i + by_j + c\right)\right]^2 \tag{8}$$

The condition on which S is minimized is that the partial derivatives of S (i.e. sum of the squares of the residuals) with respect to the constants a, b and c are equal to zero (Unwin, 1978);

$$\text{i.e.} \quad \frac{\partial S}{\partial a} = \frac{\partial S}{\partial b} = \frac{\partial S}{\partial c} = 0 \tag{9}$$

Differentiating Eq. (8) with respect to a, b and c and equate to zero,

$$\left.\begin{aligned}
\frac{\partial S}{\partial a} &= 2\sum_{\substack{i=1\\j=1}}^{N} \left[Y_{ij} - \left(ax_i + by_j + c\right)\right] \cdot (-x_i) = 0 \\
\frac{\partial S}{\partial b} &= 2\sum_{\substack{i=1\\j=1}}^{N} \left[Y_{ij} - (ax_i + by_i + c)\right] \cdot \left(-y_j\right) = 0 \\
\frac{\partial S}{\partial c} &= 2\sum_{\substack{i=1\\j=1}}^{N} \left[Y_{ij} - \left(ax_i + by_j + c\right)\right] \cdot (-1) = 0
\end{aligned}\right\} \tag{10}$$

This is implies that

$$\left.\begin{aligned}
&2\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-\left(ax_i+by_j+c\right)\right]\cdot(-x_i)=0\\
&2\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-(ax_i+by_i+c)\right]\cdot\left(-y_j\right)=0\\
&\frac{\partial S}{\partial c}=2\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-\left(ax_i+by_j+c\right)\right]\cdot(-1)=0
\end{aligned}\right\} \tag{11}$$

Dividing Eq. (11) by 2

$$\left.\begin{aligned}
&\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-\left(ax_i+by_j+c\right)\right]\cdot(-x_i)=0\\
&\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-(ax_i+by_i+c)\right]\cdot\left(-y_j\right)=0\\
&\sum_{\substack{i=1\\j=1}}^{N}\left[Y_{ij}-\left(ax_i+by_j+c\right)\right]\cdot(-1)=0
\end{aligned}\right\} \tag{12}$$

Opening the brackets,

$$\left.\begin{aligned}
&\sum_{\substack{i=1\\j=1}}^{N}\left[-Y_{ij}x_i-\left(-ax_i^2-bx_iy_j-cx_i\right)\right]=0\\
&\sum_{\substack{i=1\\j=1}}^{N}\left[-Y_{ij}y_j-\left(-ax_iy_j-by_j^2-cy_j\right)\right]=0\\
&\sum_{\substack{i=1\\j=1}}^{N}\left[-Y_{ij}-\left(-ax_i-by_j-c\right)\right]=0
\end{aligned}\right\} \tag{13}$$

Opening further, Eq. (13) becomes

$$\left.\begin{aligned}
&-\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}x_i+a\sum_{i=1}^{N}x_i^2+b\sum_{\substack{i=1\\j=1}}^{N}x_iy_j+c\sum_{i=1}^{N}x_i=0\\
&-\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}y_j+a\sum_{\substack{i=1\\j=1}}^{N}x_iy_j+b\sum_{j=1}^{N}y_j^2+c\sum_{j=1}^{N}y_j=0\\
&-\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}+a\sum_{i=1}^{N}x_i+b\sum_{\substack{i=1\\j=1}}^{N}y_jx_i+Nc=0
\end{aligned}\right\} \tag{14}$$

Taking the negative quantities to the right hand side of the equation,

$$\left.\begin{aligned}
&a\sum_{i=1}^{N}x_i^2+b\sum_{\substack{i=1\\j=1}}^{N}x_iy_j+c\sum_{i=1}^{N}x_i=\sum_{\substack{i=1\\j=1}}Y_{ij}x_i\\
&a\sum_{i=1}^{N}x_iy_j+b\sum_{j=1}^{N}y_j^2+c\sum_{j=1}^{N}y_j=\sum_{\substack{i=1\\j=1}}Y_{ij}y_j\\
&a\sum_{i=1}^{N}x_i+b\sum_{j=1}^{N}y_j+Nc=\sum_{j=1}^{N}Y_j
\end{aligned}\right\} \tag{15}$$

This can be further put in matrix form as shown in Eq. (16).

$$\begin{bmatrix} \sum_{i=1}^{N} x_i^2 & \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{i=1}^{N} x_i \\ \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{j=1}^{N} y_j^2 & \sum_{j=1}^{N} y_j \\ \sum_{i=1}^{N} x_i & \sum_{j=1}^{N} y_j & N \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} x_i \\ \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} y_j \\ \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} \end{bmatrix} \tag{16}$$

Eq. (16) is a typical form of matrix equation as can be seen in existing literature (Unwin, 1978; Davis, 2014). With Eq. (16), simultaneous equation can be applied in determining the constants a, b and c. However, with a bulk data set, this method becomes cumbersome.

Eq. (16) is equivalent to the matrix equation:

$$DE = F$$

where:

$$D = \begin{bmatrix} \sum_{i=1}^{N} x_i^2 & \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{i=1}^{N} x_i \\ \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{j=1}^{N} y_j^2 & \sum_{j=1}^{N} y_j \\ \sum_{i=1}^{N} x_i & \sum_{j=1}^{N} y_j & N \end{bmatrix}, \quad E = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} x_i \\ \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} y_j \\ \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} \end{bmatrix}$$

Hence, our matrix Eq. (16) becomes

$$DE = F \tag{17}$$

Calculating the inverse of D (i.e. D^{-1}),

$$D^{-1} = \frac{1}{|D|} \cdot adjD \tag{18}$$

where:

$adjD$ the Adjoined of D
$|D|$ the determinant of D

The determinant of D is given as

$$|D| = \begin{vmatrix} \sum_{i=1}^{N} x_i^2 & \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{i=1}^{N} x_i \\ \sum_{\substack{i=1\\j=1}}^{N} x_i y_j & \sum_{j=1}^{N} y_j^2 & \sum_{j=1}^{N} y_j \\ \sum_{i=1}^{N} x_i & \sum_{j=1}^{N} y_j & N \end{vmatrix} = \sum_{i=1}^{N} x_i^2 \left(N \sum_{j=1}^{N} y_j^2 - \sum_{j=1}^{N} y_j \sum_{j=1}^{N} y_j \right) - \sum_{\substack{i=1\\j=1}}^{N} x_i y_j \left(N \sum_{\substack{i=1\\j=1}}^{N} x_i y_j - \sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j \right)$$
$$+ \sum_{i=1}^{N} x_i \left(\sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{j=1}^{N} y_j - \sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j^2 \right) \tag{19}$$

To determine the adjoined of D, let the matrix D be represented by

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \tag{20}$$

and let c_{ij} be the entries of the matrix of cofactor of D, such that c_{11} is a cofactor of d_{11} and so on. Hence, by the definition

$$C_{11} = \left(N\sum_{j=1}^{N} y_j^2 - \sum_{j=1}^{N} y_j \sum_{j=1}^{N} y_j\right) \tag{21}$$

$$C_{12} = -\left(N\sum_{\substack{i=1\\j=1}}^{N} x_i y_j - \sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j\right) \tag{22}$$

$$C_{13} = \left(\sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{j=1}^{N} y_j - \sum_{j=1}^{N} x_i \sum_{j=1}^{N} y_j^2\right) \tag{23}$$

$$C_{21} = -\left(N\sum_{\substack{i=1\\j=1}}^{N} x_i y_j - \sum_{j=1}^{N} y_j \sum_{i=1}^{N} x_i\right) \tag{24}$$

$$C_{22} = \left(N\sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i\right) \tag{25}$$

$$C_{23} = -\left(\sum_{i=1}^{N} x_i^2 \sum_{j=1}^{N} y_j - \sum_{i=1}^{N} x_i \sum_{\substack{i=1\\j=1}}^{N} x_i y_j\right) \tag{26}$$

$$C_{31} = \left(\sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{j=1}^{N} y_j - \sum_{j=1}^{N} y_j^2 \sum_{i=1}^{N} x_i\right) \tag{27}$$

$$C_{32} = -\left(\sum_{i=1}^{N} x_i^2 \sum_{j=1}^{N} y_j - \sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{i=1}^{N} x_i\right) \tag{28}$$

$$C_{33} = \left(\sum_{i=1}^{N} x_i^2 \sum_{j=1}^{N} y_j^2 - \sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{\substack{i=1\\j=1}}^{N} x_i y_j\right) \tag{29}$$

Let C be the matrix of cofactor of D. Thus,

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{30}$$

By the definition of the adjoined of a matrix,

$$adjD = C^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \tag{31}$$

where C^T is the transpose of C.

Substituting Eq. (31) into Eq. (18),

$$D^{-1} = \frac{1}{|D|}\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \tag{32}$$

$$\therefore D^{-1} = \begin{bmatrix} \frac{C_{11}}{|D|} & \frac{C_{21}}{|D|} & \frac{C_{31}}{|D|} \\ \frac{C_{12}}{|D|} & \frac{C_{22}}{|D|} & \frac{C_{32}}{|D|} \\ \frac{C_{13}}{|D|} & \frac{C_{23}}{|D|} & \frac{C_{33}}{|D|} \end{bmatrix} \tag{33}$$

Table 1
Summary of values of equation symbols.

xi	y_j	Y_{ij}	N	x_iy_j	x_i^2	y_j^2	$Y_{ij}x_i$	$Y_{ij}y_j$
122, 720	105, 706	189, 117	16, 641	779,5 27.4	907,0 40.9	673, 155.1	1, 455, 105	1, 222, 040

Pre-multiplying Eq. (17) with D^{-1}

$$D^{-1}DE = D^{-1}F \tag{34}$$

⇒

$$IE = D^{-1}F \tag{35}$$

where $I =$ identity matrix.

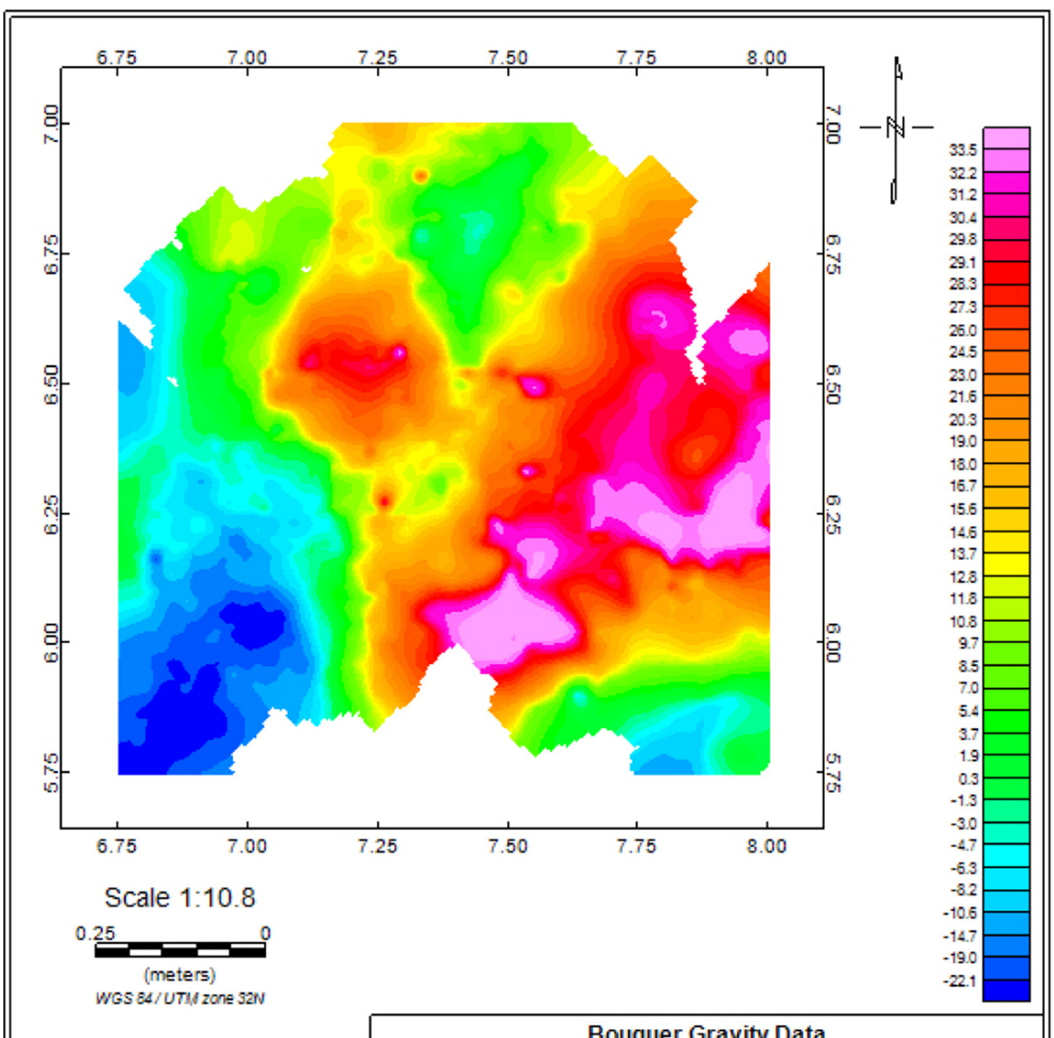

Fig. 1. Bouguer gravity map of Anambra Basin.

Substituting for E, D^{-1} and F in Eq. (35)

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{C_{11}}{|D|} & \frac{C_{21}}{|D|} & \frac{C_{31}}{|D|} \\ \frac{C_{12}}{|D|} & \frac{C_{22}}{|D|} & \frac{C_{32}}{|D|} \\ \frac{C_{13}}{|D|} & \frac{C_{23}}{|D|} & \frac{C_{33}}{|D|} \end{bmatrix} \begin{bmatrix} \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij} x_i \\ \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij} y_j \\ \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij} \end{bmatrix} \tag{36}$$

Let: $F_{11} = \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij} x_i$, $F_{21} = \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij} y_j$ and $F_{31} = \sum\limits_{\substack{i=1 \\ j=1}}^{N} Y_{ij}$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{C_{11}}{|D|} & \frac{C_{21}}{|D|} & \frac{C_{31}}{|D|} \\ \frac{C_{12}}{|D|} & \frac{C_{22}}{|D|} & \frac{C_{32}}{|D|} \\ \frac{C_{13}}{|D|} & \frac{C_{23}}{|D|} & \frac{C_{33}}{|D|} \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \end{bmatrix} \tag{37}$$

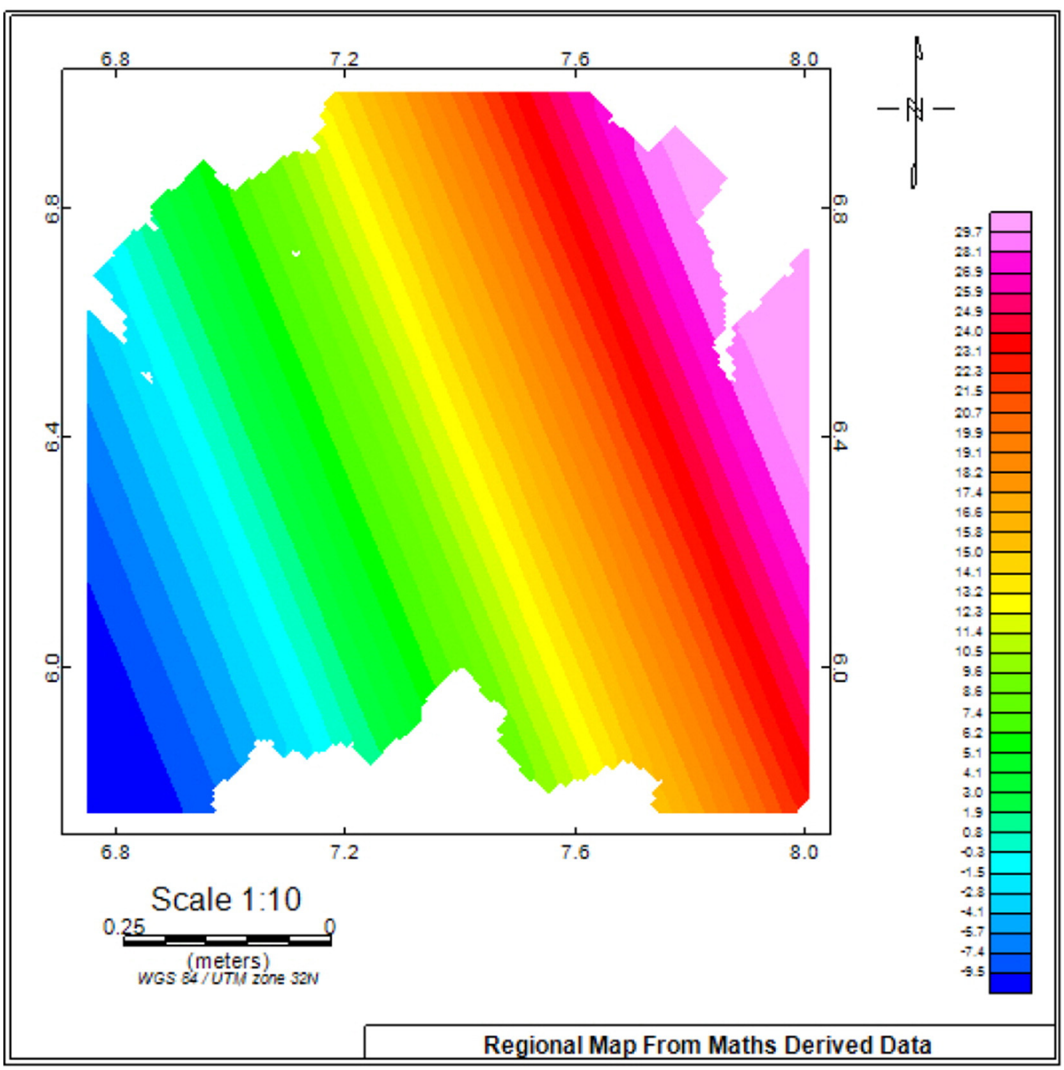

Fig. 2. Regional map of Anambra Basin.

$$\Rightarrow \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{C_{11}F_{11}}{|D|} + \frac{C_{21}F_{21}}{|D|} + \frac{C_{31}F_{31}}{|D|} \\ \frac{C_{12}F_{11}}{|D|} + \frac{C_{22}F_{21}}{|D|} + \frac{C_{32}F_{31}}{|D|} \\ \frac{C_{13}F_{11}}{|D|} + \frac{C_{23}F_{21}}{|D|} + \frac{C_{33}F_{31}}{|D|} \end{bmatrix} \quad \text{(By multiplication of matrices)} \tag{38}$$

$$\Rightarrow \quad \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{C_{11}F_{11} + C_{21}F_{21} + C_{31}F_{31}}{|D|} \\ \frac{C_{12}F_{11} + C_{22}F_{21} + C_{32}F_{31}}{|D|} \\ \frac{C_{13}F_{11} + C_{23}F_{21} + C_{33}F_{31}}{|D|} \end{bmatrix} \tag{39}$$

By equality of matrices,

$$a = \frac{C_{11}F_{11} + C_{21}F_{21} + C_{31}F_{31}}{|D|} \tag{40}$$

$$b = \frac{C_{12}F_{11} + C_{22}F_{21} + C_{32}F_{31}}{|D|} \tag{41}$$

$$c = \frac{C_{13}F_{11} + C_{23}F_{21} + C_{33}F_{31}}{|D|} \tag{42}$$

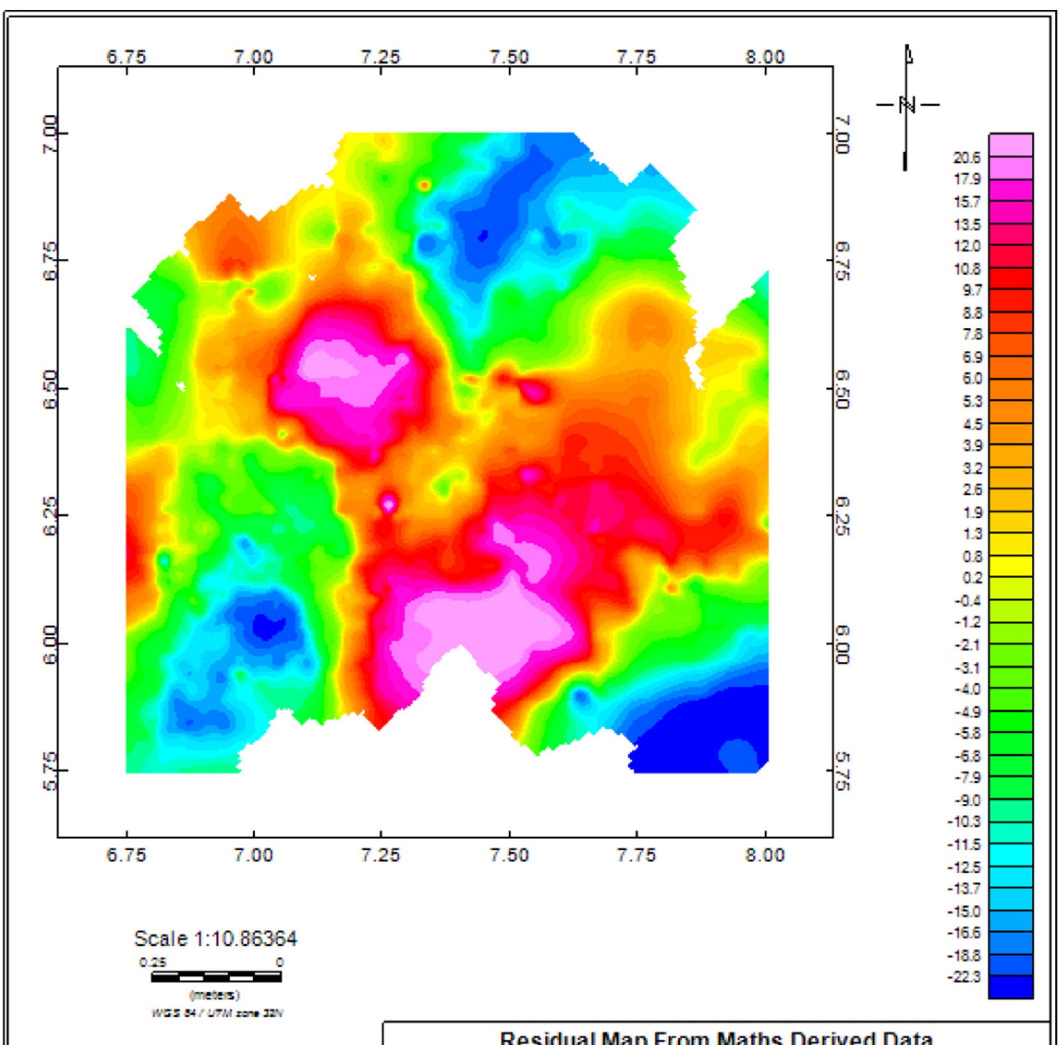

Fig. 3. Residual map of Anambra Basin.

Substituting for C_{11}, F_{11}, C_{21}, F_{21}, C_{31}, F_{31} and $|D|$ in Eq. (40),

$$\therefore a=\frac{\left(N\sum_{j=1}^{N}y_j^2-\sum_{j=1}^{N}y_j\sum_{j=1}^{N}y_j\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}x_i-\left(N\sum_{\substack{i=1\\j=1}}^{N}x_iy_j-\sum_{j=1}^{N}y_j\sum_{i=1}^{N}x_i\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}y_j+\left(\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\sum_{j=1}^{N}y_j-\sum_{j=1}^{N}y_j^2\sum_{i=1}^{N}x_i\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}}{\sum_{i=1}^{N}x_i^2\left(N\sum_{j=1}^{N}y_j^2-\sum_{j=1}^{N}y_j\sum_{j=1}^{N}y_j\right)-\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\left(N\sum_{\substack{i=1\\j=1}}^{N}x_iy_j-\sum_{i=1}^{N}x_i\sum_{j=1}^{N}y_j\right)+\sum_{i=1}^{N}x_i\left(\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\sum_{j=1}^{N}y_j-\sum_{i=1}^{N}x_i\sum_{j=1}^{N}y_j^2\right)} \tag{43}$$

Substituting for C_{11}, F_{11}, C_{21}, F_{21}, C_{31}, F_{31} and $|D|$ in Eq. (41),

$$\therefore b=\frac{-\left(N\sum_{\substack{i=1\\j=1}}^{N}x_iy_j-\sum_{i=1}^{N}x_i\sum_{j=1}^{N}y_j\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}x_i+\left(N\sum_{i=1}^{N}x_i^2-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}x_i\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}y_j-\left(\sum_{i=1}^{N}x_i^2\sum_{j=1}^{N}y_j-\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\sum_{i=1}^{N}x_i\right)\sum_{\substack{i=1\\j=1}}^{N}Y_{ij}}{\sum_{i=1}^{N}x_i^2\left(N\sum_{j=1}^{N}y_j^2-\sum_{j=1}^{N}y_j\sum_{j=1}^{N}y_j\right)\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\left(N\sum_{\substack{i=1\\j=1}}^{N}x_iy_j-\sum_{i=1}^{N}x_i\sum_{j=1}^{N}y_j\right)+\sum_{i=1}^{N}x_i\left(\sum_{\substack{i=1\\j=1}}^{N}x_iy_j\sum_{j=1}^{N}y_j-\sum_{i=1}^{N}x_i\sum_{j=1}^{N}y_j^2\right)}. \tag{44}$$

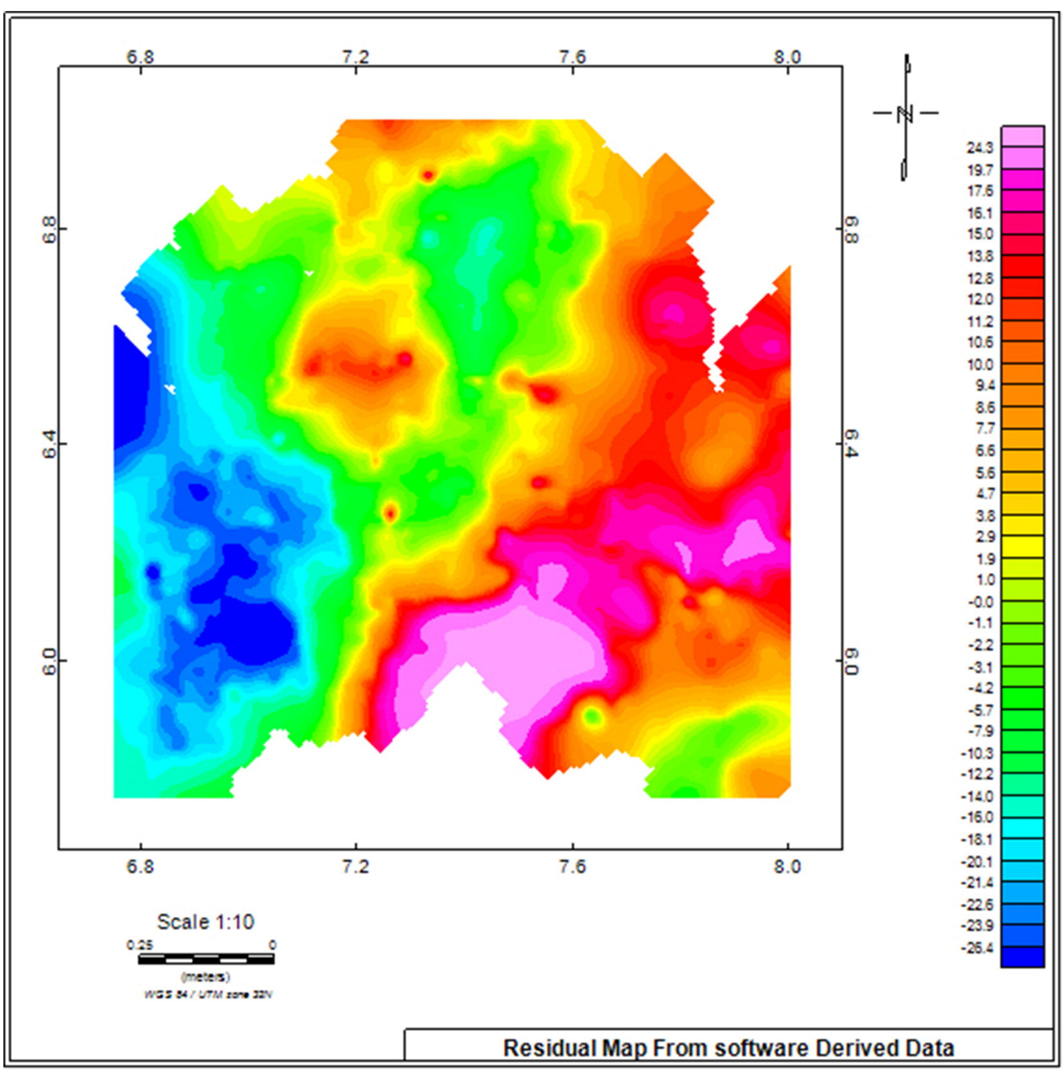

Fig. 4. Software derived residual map of Anambra Basin.

Substituting for C_{11}, F_{11}, C_{21}, F_{21}, C_{31}, F_{31} and $|D|$ in Eq. (42),

$$\therefore c = \frac{\left(\sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{j=1}^{N} y_j - \sum_{j=1}^{N} x_i \sum_{j=1}^{N} y_j^2\right) \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} x_i - \left(\sum_{i=1}^{N} x_i^2 \sum_{j=1}^{N} y_j - \sum_{i=1}^{N} x_i \sum_{\substack{i=1\\j=1}}^{N} x_i y_j\right) \sum_{\substack{i=1\\j=1}}^{N} Y_{ij} y_j + \left(\sum_{i=1}^{N} x_i^2 \sum_{j=1}^{N} y_j^2 - \sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{\substack{i=1\\j=1}}^{N} x_i y_j\right) \sum_{\substack{i=1\\j=1}}^{N} Y_{ij}}{\sum_{i=1}^{N} x_i^2 \left(N \sum_{j=1}^{N} y_j^2 - \sum_{j=1}^{N} y_j \sum_{j=1}^{N} y_j\right) - \sum_{\substack{i=1\\j=1}}^{N} x_i y_j \left(N \sum_{\substack{i=1\\j=1}}^{N} x_i y_j - \sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j\right) + \sum_{i=1}^{N} x_i \left(\sum_{\substack{i=1\\j=1}}^{N} x_i y_j \sum_{j=1}^{N} y_j - \sum_{i=1}^{N} x_i \sum_{j=1}^{N} y_j^2\right)}. \tag{45}$$

Eqs. (43), (44) and (45) are the relevant equations for the derivation of the constants a, b, and c. After the derivations of the constants, they are substituted into Eq. (6) to enable us calculate the residuals by subtracting the determined regional gravity values from the observed Bouguer gravity values.

5. Discussion

A test of the three Eqs. (43), (44), and (45) was done using a gravity data set acquired in the Anambra Basin, southeastern Nigeria, West Africa. The Anambra Basin is one of the inland basins in Nigeria. In recent years, it became an interest area due to its hydrocarbon potentials. The given data set was used to generate values for all the symbols represented in the three equations. A summary of the data set symbols is given in Table 1.

Substitution of the values in Table 1 into Eqs. (43), (44) and (45) gave the values of the constants a, b, and c respectively as represented in Eqs. (46), (47) and (48).

$$a = 29.78765775 \tag{46}$$

$$b = 12.36490103 \tag{47}$$

$$c = -286.551731 \tag{48}$$

Substituting Eqs. (46), (47) and (48) into Eq. (3), gave the regional anomaly formula as

$$\Delta g_R = 29.78765775 x_i + 12.36490103 y_j - 286.551731 \tag{49}$$

Substituting Eq. (49) into Eq. (6), the residual (Eq. (6)) becomes

$$e_{ij} = Y_{ij} - \left(29.78765775 x_i + 12.36490103 y_j - 286.551731\right) \tag{50}$$

Fig. 1 is the Bouguer gravity map of the Anambra Basin. Eqs. (49) and (50) were applied in generating the data set for plotting Figs. 2 and 3 as regional and residual gravity maps of the Anambra Basin respectively. Fig. 4 is a residual gravity map automatically generated from the Bouguer map using polynomial blog-in inside the Oasis Montaj software. A comparison of Fig. 3 with Fig. 4 proved this equation to have a better enhancing capacity than the existing software. This can be observed from both the northeast, southeast and north central portions of the residual maps.

6. Conclusion

This work has shown that separation of regional–residual anomalies during the processing of a large gravity data set is possible by using trend surface analysis, without the application of any special software. Even when computer programmes are in use, the young scholars can now appreciate the steps and activities being carried out by the computer. A comparison of this equation with existing software has shown that it has a higher enhancing capacity.

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