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Uncertainty analysis for extreme flood events in a semi-arid region

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Abstract Extreme flood events are complex and inherently uncertain phenomenons. Consequently forecasts of floods are inherently uncertain in nature due to various sources of uncertainty including model uncertainty, input uncertainty, and parameter uncertainty. This paper investigates two types of natural and model uncertainties in extreme rainfall–runoff events in a semi-arid region. Natural uncertainty is incorporated in the distribution function of the series of annual maximum daily rainfall, and model uncertainty is an epistemic uncertainty source. The kinematic runoff and erosion model was used for rainfall–runoff simulation. The model calibration scheme is carried out under the generalized likelihood uncertainty estimation framework to quantify uncertainty in the rainfall–runoff modeling process. Uncertainties of the rainfall depths—associated with depth duration frequency curves—were estimated with the bootstrap sampling method and described by a normal probability density function. These uncertainties are presented in the rainfall–runoff modeling for investigation of uncertainty effects on extreme hydrological events discharge and can be embedded into guidelines for risk-based design and management of urban water infrastructure.

Keywords Flood · Uncertainty · GLUE · Bootstrap sampling · GEV

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1 Introduction

Hydrological extreme value analysis is a key part of water resource safety assessment procedures approaches. The magnitude and frequency of hydrological extreme events such as drought and flooding are both increasing with environmental changes such as climate change [Intergovernmental Panel on Climate Change (IPCC) 2002]. In the past decades, damages caused by extreme natural hazards have drastically increased worldwide (Goldstein et al. 2003). The disaster caused by these extreme events results in an increasing demand for research on adequately modeling hydrological extreme events. Uncertainty analysis has important roles in hydrological extreme value analysis. Uncertainty is defined as a measure of imperfect knowledge or probable error that can occur during the data collection process, modeling and analysis of engineering system, and prediction of a random process. Engineering systems, such as wastewater treatment plants, soil remediation systems, water purification systems, flood control systems, are subject to uncertainty but decisions on their planning, design, operation, and management are often made without accounting for it (Sohrabi et al. 2003). There are comprehensive taxonomies of uncertainty in the literature which discuss different types and sources of uncertainty (Haimes 1998; van Asselt and Rotmans 2002; Chen and Chau 2006; Mirzaei et al. 2015; Goodarzi et al. 2012; Salarpour et al. 2013). For the purpose of this paper, it is important to recognize two basic kinds of uncertainty that are fundamentally different from each other: natural and epistemic uncertainty.

Natural uncertainty refers to quantities that are inherently variable over time, space, or populations of individuals or objects. Variability exists, for example, in the amount of annual rainfall in consecutive years, in the clay content of a field, or in the body weight of adults. Epistemic uncertainty is related to our ability to understand, measure, and describe the system under study. For example, if we use a mathematical model to describe a system, epistemic uncertainty may consist of model (or structural) uncertainty and parameter uncertainty (Cullen and Frey 1999).

The kinematic runoff and erosion model (KINEROS) was used for rainfall–runoff simulation. The uncertainty analysis methods selected for use in KINEROS must be able to handle propagation of uncertainty and variability of the model input parameters, taking into account distributions of parameter uncertainty. The method will be used to provide uncertainties of model outputs in terms of distributions of model outputs, joint distributions of model inputs and outputs. For performing a thorough uncertainty and variability analysis, the generalized likelihood uncertainty estimation (GLUE) has a number of advantages over other methods (Xu et al. 2010). The GLUE is used for assessing the uncertainty associated with model predictions, which assumes that due to the limitations in model structure, data and calibration scheme, many different parameter sets can make acceptable simulations.

The GLUE technique introduced by Beven and Binley (1992) is an innovative uncertainty method that is often employed with environmental simulation models. There are now over 500 citations to their original paper which illustrate its applicability and accurateness [e.g., Lindblom et al. (2011), Vázquez and Feyen (2010), Callies et al. (2008), Blasone et al. (2008b), Beven et al. (2007, 2008), Mantovan and Todini (2006), Mantovan et al. (2007), Mirzaei et al. (2015)]. The primary advantages of this method are the reduction in the number of simulations required, the ability to use different ways of specifying parameter distributions, the ability to handle very complex models, and the propagation of variability, uncertainty, and parameter dependencies through the model that are reflected in the distributions of model outputs. The bootstrap method, a simple nonparametric technique, is proposed in this paper as it is simple to describe and easy to implement. The bootstrap is a technique for determining the accuracy of statistics in circumstances in which confidence intervals cannot be obtained analytically or when an approximation based on the limit distribution is not satisfactory (Efron and Tibshirani 1993; Davison and Hinkley 1997). Bootstrap techniques have become very popular in many areas of environmental sciences, including frequency analysis in climatology and hydrology (Dunn 2001; Hall et al. 2004; Ames 2006; Kyselý and Beranová 2009; Twardosz 2009; Fowler and Ekström 2009).

This study is motivated by observed and modeled increases in critical design precipitation events and seeks to better quantify the magnitude and uncertainty of extreme precipitation–runoff events. It is the intent of this research to increase our understanding of the inherent uncertainties in statistical frequency analysis of extreme precipitation events and the uncertainties as a result of the rainfall–runoff modeling. The main objective of this study is the uncertainty analysis of extreme hydrological events in semi-arid regions. In order to pursue the main objective, the specific objectives of the study were: (1) to quantify the uncertainties in frequency analysis of extreme rainfall events which are associated with depth duration frequency curves (DDF), (2) to quantify the uncertainty in the rainfall– runoff modeling process using the calibration scheme carried out under the GLUE framework, and (3) to represent these uncertainties in rainfall–runoff modeling for uncertainty effects in discharge and volumes of extreme hydrological events. This research work was demonstrated and applied to the Zayanderood basin in central Iran.

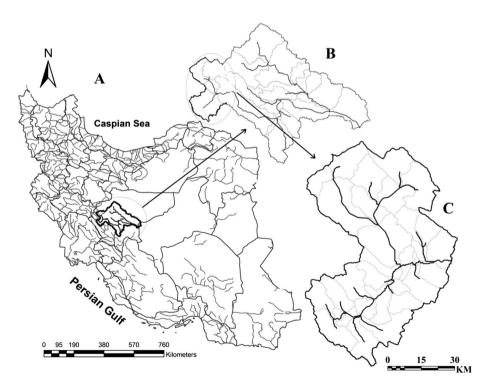


Fig. 1 Location map of Zayanderood basin in Iran (a), location map of study area in Zayanderood basin (b) and map of study area (c)

2 Study area

The west of the Zayanderood River catchment in central Iran, shown in Fig. 1, is used throughout this study for the demonstration of the methodologies and the models developed in this study. This area was selected because it covers the main source of the streamflow in the Zayanderood River and it has a reasonably dense network of rain-gauge and runoff-gauge stations. This semi-arid region has an annual average precipitation of 611 mm and an annual average temperature of 11 °C; there is a seasonal distribution of precipitation with the wet season being in autumn, winter and spring, and the dry season in summer. The four selected stations have more than 50 years of daily rainfall data records.

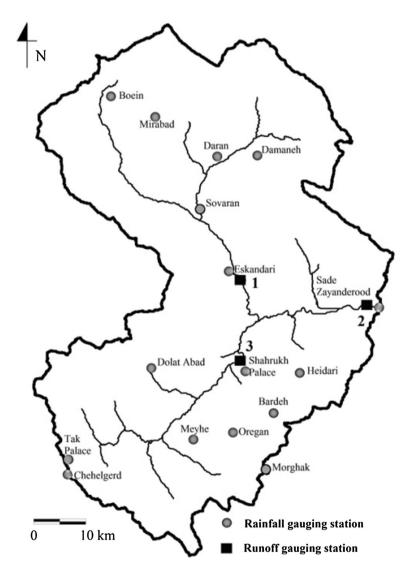


Fig. 2 Locations of the rainfall and runoff gauging stations in the study area (after Mirzaei et al. 2013a)

All the rainfall stations are depicted in Fig. 2. Table 1 lists the rainfall stations used in this study.

3 Modeling uncertainty in DDF curves

Uncertainty in DDF curves which is usually disregarded in view of the difficulties associated in assigning a value to it should be considered in the design of hydraulic structures. To investigate the uncertainty in DDF curves, the bootstrapping method is used to calculate the confidence bands of the DDF curves. Only the uncertainty due to the estimation of the generalized extreme value (GEV) distribution parameters and the associated sampling errors were evaluated in this study.

One of the simpler and frequently used models that is popularly utilized in statistics is the classical regression model

$$Y = X\beta + \epsilon$$

where $Y = (Y_1, ..., Y_n)^t$ represents an observational vector of length n, X is a $n \times p$ known matrix of explanatory variables, and β is a vector of unknown regression coefficients of length p that characterizes the relationship between observations and explanatory variables.

Classically, the vector ϵ is assumed to be a zero-mean Gaussian vector. In this study, X and β were defined as:

$$X = \begin{pmatrix} 1 \ln D_1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 \ln D_5 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

where *D* is rainfall duration for 24, 48, 72, 96, and 120 h, and β_0 and β_1 are regression coefficients of GEV parameters. The generalized least squares method was used to estimate the regression coefficients β_0 and β_1 .

Relations of the GEV parameters as a function of duration D (hours) were used to construct rainfall DDF curves. Now that the GEV parameters are described as a function of D, rainfall DDF curves are constructed by substituting these relationships into below equation, so that the DDF curves are given by:

Station name	National code	Study code	Data period from–to	Latitude (N)	Longitude (E)	Elevation (m)
Chelgerd	42001	S 1	1954–2009	50.13	32.45	2324
Damaneh	42004	S2	1954-2009	50.48	33.02	2300
Shahrukh Palace	42003	S 3	1958–2009	50.47	32.65	2098
Sade Zayanderood	42007	S 4	1955–2009	50.78	32.72	1990

Table 1 Selected stations, their record length, latitude and longitude

$$\hat{x}(T) = \exp\left(\hat{\beta}_{0\xi} + \hat{\beta}_{1\xi}\ln D\right) \times \left(1 + \left(\hat{\beta}_{0\gamma} + \hat{\beta}_{1\gamma}\ln D\right) \frac{\left\{1 - \left[-\ln\left(1 - T^{-1}\right)\right]^{\hat{k}_{\text{GLS}}}\right\}}{\hat{k}_{\text{GLS}}}\right)$$

4 Monte Carlo sampling

Monte Carlo is a method for the calculation of parameter confidence intervals; two Monte Carlo-based approaches are commonly seen in literatures: importance sampling and Markov chain Monte Carlo (MCMC) simulation (e.g., Bates and Campbell 2001; Gallagher and Doherty 2007). Importance sampling is a technique for randomly sampling from a probability distribution and was implemented in GLUE by Beven and Binley (1992). MCMC is one of the most important numerical techniques for creating a sample from the posterior distribution, which has been widely used in hydrological modeling to quantify parameter uncertainties (e.g., Makowski et al. 2002). Its underlying rationale is to set up a Markov chain to simulate the true posterior distribution by generating samples from a random walk. An obvious advantage of this method is that it does not require linearity assumptions in model or even that model outputs do not need to be differentiable with respect to parameter values (Gallagher and Doherty 2007). Because of its robust performance, MCMC is often used to assess parameter uncertainties in combination with GLUE

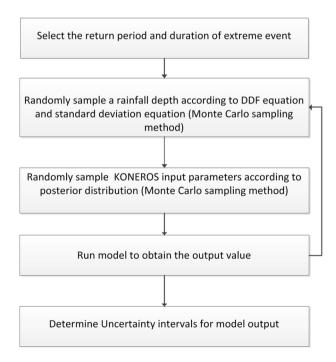


Fig. 3 Flowchart to determine uncertainty intervals

or Bayesian inference by estimating a probability density for model parameters conditioned on observations. Blasone et al. (2008) found that using a MCMC sampling scheme coupled with GLUE significantly improves the efficiency and effectiveness of the methodology of GLUE.

5 Uncertainty analysis in modeling of extreme rainfall–runoff events

The input data and parameter set that users provide are usually the representation of the average condition of a study site (for example, rainfall depth and hydraulic conductivity). However, assigning a value to that representative variable inherently involves a certain degree of uncertainty, which will directly affect the level of uncertainty of the model prediction.

The structure of uncertainty analysis for extreme rainfall–runoff events is organized as illustrated in Fig. 3. The process that is shown in Fig. 3 was done for five return periods and five rainfall durations. Selected rainfall durations for this study are 24, 48, 72, 96, and 120 h. The number of samples for simulation was limited by the simulation time-consuming. Model run time was about 8 min for one simulation, so total time for the exercise was about 130 h for 1000 samples. In this study, 1000 samples were selected for each return period and rainfall duration.

After running KINEROS for all sample sets, the model output (runoff) was analysis for each event and the predicted runoff and the upper and lower uncertainty were determined. The KINEROS was applied and developed for 1000 input parameter sets to generate runoff hydrograph data and uncertainty boundaries. All output hydrographs data were sorted according to maximum peak discharge. Then for determination of 95 % probability prediction uncertainty (95PPU), all outputs between 25 and 975 were selected for analysis. The next step was provided for output hydrographs and 95PPU band for each event.

D (h)	S 1			S2			
	Ê	Ŷ	ĥ	ŝ	Ŷ	ĥ	
24	34.90 (2.47)	0.552 (0.002)	0.019 (0.024)	8.45 (1.11)	1.049 (0.003)	0.140 (0.016)	
48	56.43 (9.49)	0.601 (0.005)	0.015 (0.015)	13.21 (1.48)	1.060 (0.005)	0.005 (0.012)	
72	81.52 (11.83)	0.555 (0.004)	0.014 (0.014)	22.37 (3.73)	1.074 (0.010)	0.120 (0.014)	
96	92.28 (16.27)	0.543 (0.006)	0.021 (0.020)	24.72 (5.99)	1.070 (0.011)	0.080 (0.015)	
120	118.29 (20.87)	0.600 (0.003)	0.016 (0.027)	30.83 (6.18)	1.061 (0.016)	0.100 (0.068)	
<i>D</i> (h)	S3			S4			
	ŝ	Ŷ	ĥ	ξ	Ŷ	ĥ	
24	11.27 (1.16)	1.117 (0.002)	0.181 (0.018)	9.86 (1.20)	0.698 (0.002)	0.088 (0.018)	
48	17.34 (2.21)	1.075 (0.004)	0.159 (0.022)	14.47 (2.42)	0.671 (0.004)	0.141 (0.022)	
72	22.53 (2.40)	1.129 (0.011)	0.145 (0.042)	20.05 (2.23)	0.638 (0.011)	0.118 (0.042)	
96	28.63 (3.88)	1.197 (0.003)	0.170 (0.045)	23.36 (3.88)	0.701 (0.003)	0.101 (0.045)	
120	34.87 (4.46)	1.124 (0.020)	0.186 (0.045)	30.50 (4.46)	0.715 (020)	0.119 (0.045)	

Table 2 Estimated GEV parameters for D = 24, 48, 72, 96 and 120 h in all stations, standard deviations are estimated with the bootstrap and given between brackets (Mirzaei et al. 2013b)

6 Results and discussion

6.1 GEV parameters estimation

Study area has 16 rainfall gauge stations (Fig. 2). For statistical analysis, stations that have more than 50 years of data are studied. According to Table 1, only four stations have longer than 50-year data. These stations included Damaneh, Shahrukh Palace, Chelgerd and Sade Zayanderood that have 55, 51, 55, and 54 years recorded data, respectively. The GEV parameters were estimated for 24-, 48-, 72-, 96-, and 120-h rainfall duration. According to the equations in the GEV regression model, the regression coefficients were estimated for the 10⁴ bootstrap samples. Table 2 shows the averages and standard deviations of the regression coefficients. In this paper, ξ , α and k are the location, scale, and shape parameters, respectively, and $\lambda = \alpha/\xi$. For k, the estimate of slope β_1 was approximately zero for most samples (for more information, refer to Mirzaei et al. 2013b).

6.2 DDF curves uncertainty estimation

The bootstrap method was employed to assess this uncertainty. This method considers only the uncertainty due to the estimation of the GEV parameters and sampling errors. For each of the 10^4 bootstrap samples, the relations between the GEV parameters and duration were re-estimated using generalized least squares, so that 10^4 DDF curves could be constructed. As mentioned before, DDF curves are constructed by equation, so that the DDF curves are given by (Mirzaei et al. 2013b):for Station S1 is

$$\hat{x}(T) = \exp(3.2149 + 0.3125 \ln D) \\ \times \left(1 + (0.5699 + 0.0009 \ln D) \frac{\left\{ 1 - \left[-\ln(1 - T^{-1}) \right]^{0.0164} \right\}}{0.0164} \right)$$

for Station S2

$$\hat{x}(T) = \exp(1.1391 + 0.3031 \ln D) \\ \times \left(1 + (3.0658 + 0.00089 \ln D) \frac{\left\{ 1 - \left[-\ln\left(1 - T^{-1}\right) \right]^{0.0857} \right\}}{0.0857} \right)$$

for StationS3

$$\hat{x}(T) = \exp(1.625 + 0.4045 \ln D) \times \left(1 + (1.1419 + 0.0009 \ln D) \frac{\left\{ 1 - \left[-\ln(1 - T^{-1}) \right]^{0.1659} \right\}}{0.1659} \right)$$

for Station S4

$$\hat{x}(T) = \exp(1.453 + 0.4042 \ln D) \times \left(1 + (0.6725 + 0.0034 \ln D) \frac{\left\{ 1 - \left[-\ln(1 - T^{-1}) \right]^{0.1105} \right\}}{0.1105} \right)$$

By choosing a return period *T*, the rainfall depth *x* (mm) can be plotted as a function of duration *D* using the above equations. Figure 6 presents the DDF curves for T = 5, 10, 20, 50, and 100 years. The curves show a strong increase in rainfall depth with *D*.

For each DDF curve, the rainfall depths were derived for durations between 24 and 120 h in steps of 1 h (Fig. 4).

6.3 Uncertainty analysis for long duration rainfall events

In this study, the flowchart in Fig. 3 was followed for uncertainty analysis. Return periods of 5, 10, 20, 50, and 100 years were selected for understanding uncertainty in future Zayanderood River stream flow. Stream flow hydrographs for each event were obtained after 1000 simulations of rainfall–runoff model using Monte Carlo sampling method. This study used irreversible MCMC algorithms for efficient sampling from the posterior distribution on the input model parameter space and probability density functions of rainfall DDF curves. Then, the uncertainty band, the space between upper and lower of 95PPU, was determined (Fig. 5).

To analyze the uncertainty in peak discharge, the lower and upper uncertainty bands are plotted in Fig. 6. The peak discharge value and uncertainty band increased with increasing return periods.

6.4 Uncertainty in peak discharge of extreme runoff events

Figure 7 presents values of peak surface runoff volumes for different rainfall depths and durations. The figure shows that both the upper and lower uncertainty bands changed negatively with the increase in rainfall duration. In fact, model accuracy in discharge calculation grows by increasing rainfall duration.

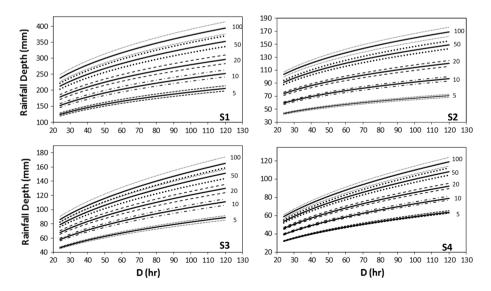


Fig. 4 Rainfall DDF curves (*solid lines*) and 95 % prediction uncertainty (*dashed lines*) for the return periods of 5, 10, 20, 50, and 100 years at stations S1, S2, S3, and S4 (Mirzaei et al. 2013b)

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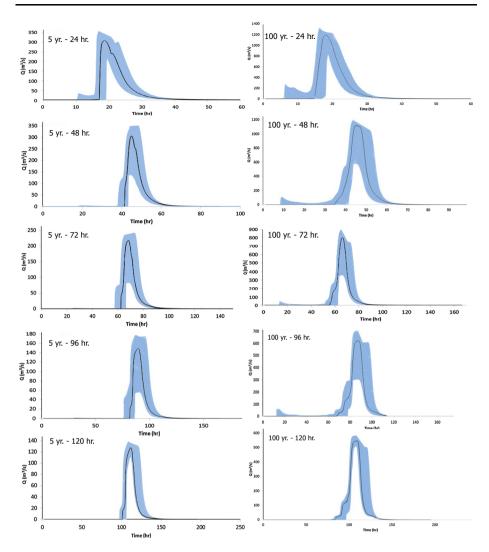


Fig. 5 Storm hydrographs of 24-, 48-, 72-, 96-, and 120-h rainfall and 5- and 100-year return period, 95 % prediction uncertainty (*shaded area*)

It is notable that peak flow rates for each return period decreased with the increase in duration. For example, peak discharge values of 5-year return period and 24-h rainfall duration were 307 m^3 /s, reducing to 127 m^3 /s for 5-year return period and 120-h rainfall. This change is very much obvious according to the concept of hydrograph This can be explained with the S-curve concept where it is learned that after certain time of a continuous rainfall on a catchment, runoff flow rate will reach a constant value. After that, flow will no more increase even though rainfall duration is further increased. Before reaching this constant flow condition, peak flow rate will obviously increase with the increase in rainfall duration but at a reduced rate. In other words, peak flow rate of effective rainfall will decrease with the increase in duration.

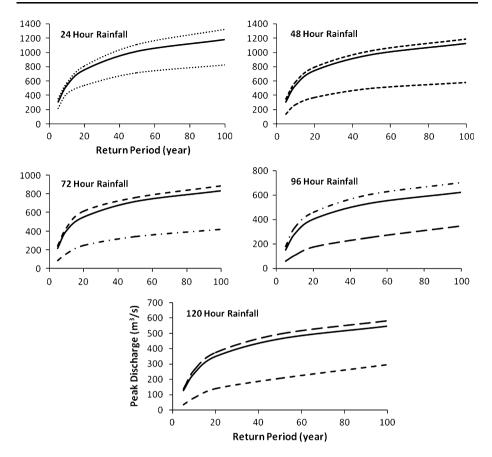


Fig. 6 Peak discharge variation (solid lines) and 95 % confidence bands (dashed lines)

For a given return period, upper uncertainty band and lower uncertainty band were different in different rainfall durations. These results indicate that there is an influence of rainfall duration on uncertainty boundaries although no specific trend was identified.

7 Conclusions

This paper studied the uncertainty in rainfall and input parameters of the KINEROS model which affect the model output (runoff). Uncertainty in extreme rainfall was investigated over various rainfall durations and return periods. Extreme rainfall events in the Zayanderood basin for rainfall durations of between 24 and 120 h were studied. Since regional variability and extreme rainfall amounts were not independent, extreme rainfall analysis was carried out for four main stations separately. GEV parameters of this time series were estimated with the method of L-moments. Standard deviations and correlations of estimated GEV parameters were obtained with bootstrapping. To take into account the correlation between estimated GEV parameters for different durations, the generalized least squares method was used to describe the variation of these parameters as a function of

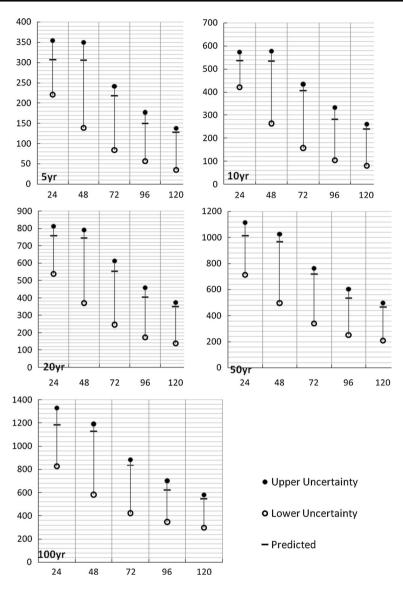


Fig. 7 Predicted and uncertainty band of peak discharge for five different return periods and five different rainfall durations (dimension of vertical axis is m^3/s)

duration. The relations were used to construct rainfall DDF curves. Finally, uncertainties in DDF curves, due to sampling variability, were quantified with the bootstrap and described with a normal distribution.

The highlighting conclusions of the study are: (1) stream hydrographs were derived using Monte Carlo simulation for the KINEROS model. The study revealed that for peak discharge and stream volume, the upper and lower band of uncertainty increased with increasing return periods. (2) The amounts of uncertainty in peak discharge reduced for all return periods with increasing rainfall duration. (3) It is concluded that for extreme events simulation and using appropriate results for designing water resources projects, the knowledge of input data uncertainty is needed. In fact, without any uncertainty analysis, the peak discharge and stream flow volume of extreme events may be more than that has been predicted by model in certain condition.

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