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Optimization of dewatering systems for mineral processing

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ABSTRACT

Water is widely used as a solvent in the mining industry and is employed in hydrometallurgical processes and for mineral concentration. Because of the global increase in metal production, the demand for water, including fresh water, is expected to increase continually. In arid and semi-arid regions such as northern Chile, the scarcity of fresh water has led to increased dependence on other sources such as sea water and triggered efforts towards optimization of the use of fresh water.

In copper concentration plants, approximately 40–60% of the total amount of water lost is retained in slurries in the tailings. In this paper, we present a method for optimizing the design of dewatering systems that employ hydrocyclones and thickeners. Mathematical models were generated to determine the maximum water recovery rate and the corresponding system structure for given equipment sizes, and to determine the minimum cost of the equipment and the corresponding system structure for given water recovery rates. The models were based on mixed integer nonlinear programming. Several case studies were performed. The model predictions were consistent with the results of an experimental study of an actual dewatering system in a copper concentrator plant.

1. Introduction

Water is commonly employed in most material processing industries and is one of the most important resources in metallurgical processes. However, industrial processes that involve the use of water are being subjected to increasingly stringent environmental regulations with respect to effluent discharge. There is a growing demand for fresh water because water is scarce in several countries and is a critical commodity in some parts of the world. These issues have increased the need for improved water management and wastewater minimization (Klemes, 2012).

In the mining industry, water is used in hydrometallurgical and mineral concentration processes. For example, in the concentration by flotation process, ore particles are reduced in size to a specific range in order to separate the species of value. At this stage, the mineral is mixed with water and wet milled to an optimal granulometry to achieve flotation. The flotation process yields two products, a concentrate and tail streams, that are sent to dewatering systems for water recovery. These systems usually include thickeners, hydrocyclones, and filters.

Water scarcity in arid and semi-arid mining areas has become one of the most important issues for these regions, as water is essential for the development of all economic activities, environmental care, and the quality of community life. The mining industry assigns vital importance to the rational and efficient use of water in its operations and has taken actions to optimize its consumption through best water management practices and the introduction of improved technologies (Usher and Scales, 2005). For example, in the Antofagasta region of Chile, the water consumption of the mining industry corresponds to 60% of the total consumption, and is projected to increase to 70% by 2020. Further, in Chile, the water consumption of the copper mining industry in 2016 will be 576.2 Mm^3; this represents an increase of 47% over the consumption level in 2012. As a result, several mining companies have started to use seawater in their operations and have also improved their water recycling systems (Johnson, 2003).

The primary technique involved in these dewatering processes is sedimentation, which is the settling of a suspension of particles in a fluid under the effect of an external force, which may be gravity, centrifugal force, or any other body force (Concha, 2001). The purpose of the dewatering system is to increase the concentration of the solid discharge removed from the pulp in order to increase the amount of water recovered.

Owing to several factors, efforts are being made to increase the efficiency of the recycling of 'used' water (Rao and Finch, 1989): (1)

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mineral processing plants can consume a large proportion of the local water resources, and this can reduce the amount of water available for other uses; (2) the collection and transportation of fresh water is usually expensive and affects the operating costs of the mining industry; and (3) the effluents from mineral processing operations are potentially detrimental to the environment because these streams contain both suspended solids as well as a number of dissolved polluting toxic chemicals, in addition to flotation agents and their degradation products.

The objective of this study was to develop a method for designing dewatering systems that allow for the maximum water recovery given a set of equipment or allow for the minimum equipment cost given the water recovery rates. This method creates a set of alternative based on hierarchical superstructures from which the structure most suited for the process in question can be selected. The problem is represented mathematically using models for these superstructures; these models are based on the principle of material balance, and equations representing the behaviors of the hydrocyclones and thickeners are employed. Finally, two objective functions are defined, namely, to optimize water recovery and to reduce the system costs.

2. Problem statement

In this section, we describe a mixed integer nonlinear programming (MINLP) model to optimize water recovery from slurry. The model/problem is labelled as P1 and is solved to determine the maximum amount of water that can be recovered from a tailings separation circuit for a specified set of equipment. This can be considered to be a retrofit problem. A second MINLP model/problem, which is referred to as P2, is used to determine the minimum cost of a dewatering system given a desired rate of water recovery. This can be considered a design problem. The dewatering system configuration and stream flow rates are calculated for both problems (i.e., for P1 and P2). For P2, the equipment design is also determined. These models/problems are independent of one another and can be used as per the objectives of the designer.

The superstructure of the water recovery system comprises a hydrocyclone system and a thickener system that along with two dividers and three mixers constitutes a set of alternatives from which the optimum system structure is selected. This superstructure is known as the overall superstructure (OS). The OS comprises a set of nine dewatering structures (Fig. 1) that correspond to various combinations of the dividers (i.e., three options for the feed slurry stream and three options for the hydrocyclone system overflow stream).

Each system in the OS (Fig. 1) has an internal superstructure. The internal superstructures of the hydrocyclone system and the thickener system are illustrated in Figs. 2 and 3, respectively. The combinatorial nature of the problem results in an exponential increase in the number of alternatives with an increase in the number of hydrocyclones and/or thickeners. Thus, the method requires that the number of hydrocyclone and thickeners employed be predefined.

Equations for the particles, water flow, and slurry were developed, including mass balance expressions and unit operation models. The resulting MINLP model for problem P1 can be formulated as follows:

$$\begin{aligned}&\text{Maximize}\quad c^T x\\&s.t.\quad h(x,y)=0\\&g(x,y)\leqslant 0\\&x\in R^n, y\in\{0,1\}^m.\end{aligned}$$

The MINLP model for problem P2 can be formulated as follows:

$$\begin{aligned}&\text{Minimize}\quad f(x,y,d)\\&s.t.h\quad (x,y,d)=0\\&g(x,y,d)\leqslant 0\\&x\in R^n, d\in R^p, y\in\{0,1\}^m,\end{aligned}$$

where y is a binary vector that denotes the rejection (i.e., $y = 0$) or acceptance (i.e., $y = 1$) of a particular alternative solution; x represents the operating variable (e.g., the mass flow rate); d denotes the vector of design variables that represent the sizes of the process units, (e.g., the diameters of the thickeners); and h and g represent the various equality and inequality constraints, such as the mass balance relations and the equipment design models, respectively. The objective function for P1 is the total amount of water recovered, whereas that for P2 is the minimum equipment cost.

2.1. Overall dewatering superstructure model

The mass balance relation for the overall superstructure can be expressed as

$$\sum_{s\in S^{in}_{e,s}} W_{s,c} - \sum_{s\in S^{out}_{e,s}} W_{s,c} = 0 \qquad \forall c\in C,\quad e\in E \tag{1}$$

Eq. (1) states that the input and output flows of each component in the hydrocyclone system, the thickener system, the mixers, and the splitters must be equal. In the equation, $W_{s,c}$ is the mass flow rate of particles of different sizes and the pulp volumetric flow

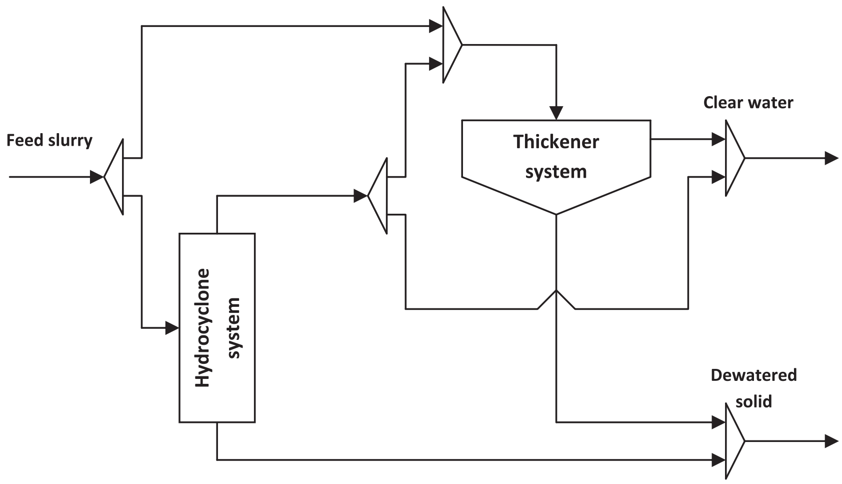

Fig. 1. Dewatering overall superstructure (OS).

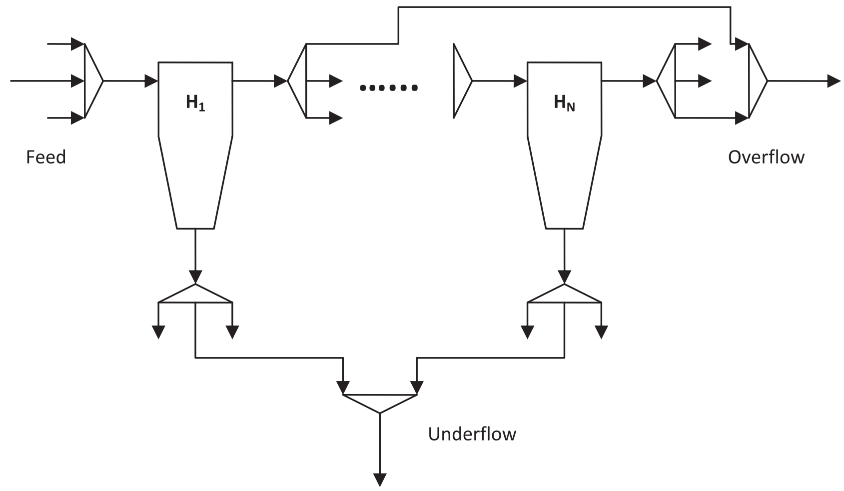

Fig. 2. Superstructure of hydrocyclone system.

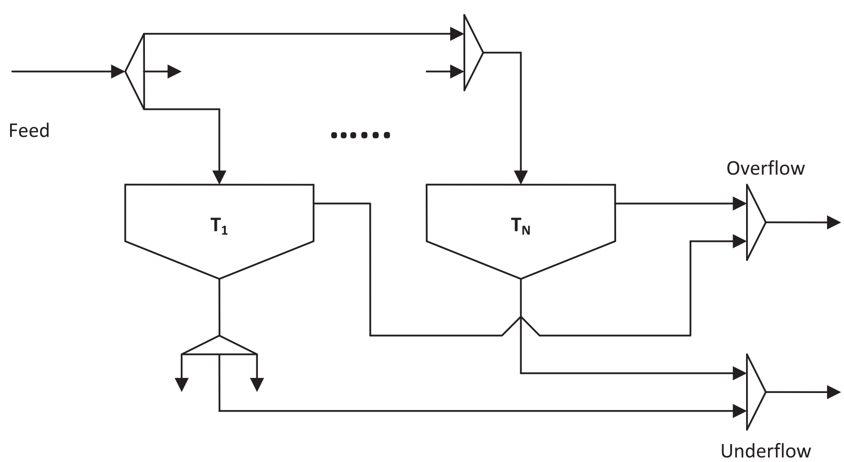

Fig. 3. Superstructure of thickening system.

rate in the overall superstructure; $S^{in}_{e,s} = \{(e,s)/s$ is an input stream $s \in S$ to the equipment $e \in E\}$; and $S^{out}_{e,s} = \{(e,s)/s$ is an output stream $s \in S$ to the equipment $e \in E\}$.

The mass balance relations for the dividers are bilinear expressions because the proportion in which they divide the input stream is not known. In this work, the division is represented by a disjunctive model in order to avoid such bilinear expressions. That is to say,

$$\underset{j \in J}{\vee} \begin{bmatrix} y_{e,j} \\ W_{s_1,c} \geqslant n_j \cdot W_{s,c} \\ W_{s_2,c} \geqslant (1-n_j) \cdot W_{s,c} \end{bmatrix} \quad \forall e \in E_D, \quad c \in C, \quad s, s_1, s_2 \in S_D \tag{2}$$

In Eq. (2), $\vee$ is the OR operator that applies to a set of disjunctive terms J. Eq. (2) discretises the decision of flow splitting into a set of J alternatives, each of which is defined by the parameter n_j. In the equation, $y_{e,j}$ is a binary variable representing the various disjunctions available and s is the stream whose flow is divided into streams s_1 and s_2. The parameter n_j is the fraction of the stream s that is sent to s_1. $S_{D1} = \{(s,s_1)/s$ is split into $s_1\}$ and $S_{D2} = \{(s,s_2)/s$ is split into $s_2\}$. Eq. (2) can be written using big M method (Raman and Grossman, 1994) as follows:

$$W_{s,c} = W_{s_1,c} + W_{s_2,c} \qquad \forall (s,s_1) \in S_{D1}, \quad (s,s_2) \in S_{D2}, \quad c \in C \tag{3}$$

$$W_{s_1,c} \geqslant n_j \cdot W_{s,c} - M \cdot (1 - y_{e,j}) \qquad \forall (s,s_1) \in S_{D1}, \quad c \in C, \quad j \in J, \quad e \in E_D \tag{4}$$

$$W_{s_2,c} \geqslant (1-n_j) \cdot W_{s,c} - M \cdot (1 - y_{e,j}) \qquad \forall (s,s_2) \in S_{D2}, \quad c \in C, \quad j \in J, \quad e \in E_D \tag{5}$$

$$\sum_{j \in J} y_{e,j} = 1 \qquad \forall e \in E_D \tag{6}$$

Note that the 0–1 variable $y_{e,j}$ is introduced to denote the disjunction j in J that is true ($y_{e,j} = 1$). The constraint in Eq. (6) only allows one choice of $y_{e,j}$. Eqs. (4) and (5) introduce on the right-hand side a big parameter, M, which renders the inequality redundant if $y_{e,j} = 0$ and forces the inequality if $y_{e,j} = 1$. Note that M is the upper bound of the variable $W_{s,c}$.

2.2. Hydrocyclone system superstructure

Equations similar to Eqs. (1)–(6) must be satisfied in the case of the hydrocyclone system:

$$\sum_{s \in S^{in}_{He,s}} W^{E_H}_{s,c} - \sum_{s \in S^{out}_{He,s}} W^{E_H}_{s,c} = 0 \qquad \forall c \in C, \quad e \in E_H \tag{7}$$

Eq. (7) is a mass balance equation for each piece of equipment, where $W_{s,c}^{E_H}$ is the mass flow rate for the particles of different sizes and the pulp volumetric flow rate in the hydrocyclone system superstructure, $S_{H_{e,s}}^{in} = \{(e,s)/s$ is an input stream $s \in S_H$ to the equipment $e \in E_H\}$, and $S_{H_{e,s}}^{out} = \{(e,s)/s$ is an output stream $s \in S_H$ to the equipment $e \in E_H\}$.

$$\bigvee_{j \in J_H} \begin{bmatrix} y_{He,j} \\ W_{s_1,c}^{E_H} \geqslant n_{H_j} \cdot W_{s,c}^{E_H} \\ W_{s_2,c}^{E_H} \geqslant (1-n_{H_j}) \cdot W_{s,c}^{E_H} \end{bmatrix} \quad \forall e \in E_H, \quad c \in C, \quad s, s_1, s_2 \in S_H \tag{8}$$

Eq. (8) is analogous to Eq. (2) and discretises the decision of flow splitting into a set of alternatives J_H, each of which is defined by the parameter n_{Hj}. In Eq. (8), $y_{H_{e,j}}$ is a binary variable representing the various disjunctions available and s is the stream whose flow is divided into streams s_1 and s_2. The parameter n_{Hj} is the fraction of the stream s that is sent to s_1. Eq. (8) can be written using the big M method (Raman and Grossmann, 1994) with equations similar to Eqs. (4)–(6); for the sake of simplicity, these equations are not shown here.

The mass balance equations for the hydrocyclone can be written using the following set $S_e^E = \{(s, s_o)/s, s_o \in S_e^E$, where s_o is the hydrocyclone $e \in E_H$ overflow stream that is fed by the stream $s\}$. That is to say,

$$W_{s_o,t}^{E_H} = W_{s,t}^{E_H} \cdot Ss_{e,t} \qquad \forall (s_o, s) \in S_e^E, \quad t \in T, \quad e \in E_H \tag{9}$$

Eq. (9) allows one to calculate the flow rate of the hydrocyclone overflow stream when the inlet flow rate $W_{s,t}^{E_H}$ is known and the selectivity function $Ss_{e,t}$ (Braun and Bohnet, 1990) and (Svarovsky, 1994) is represented by the following equation:

$$Ss_{e,t} = Cc_{e,t} + Rf_{e,p} \cdot (1 - Cc_{e,t}) \qquad \forall e \in E_H, \quad t \in T, \quad p \in P \tag{10}$$

where $Cc_{e,t}$ represents the classification function (Plitt, 1971) and $Rf_{e,p}$ is the recovery from the hydrocyclone underflow (Tarjan, 1961). $Cc_{e,t}$ and $Rf_{e,p}$ can be calculated as follows:

$$Cc_{e,t} = 1 - Exp\left[-0.693 \cdot \left(\frac{d_t}{d50_e}\right)^{1.93}\right] \qquad \forall e \in E_H, \quad t \in T \tag{11}$$

$$Rf_{e,p} = 1 - \frac{1}{1 + A \cdot \left(\frac{Du_e}{Do_e}\right)^B} \qquad \forall e \in E_H, \quad p \in P \tag{12}$$

where d_t is the diameter of the particle t and $d50_e$ is the cut size of the hydrocyclone e for the classification function $Cc_{e,t}$. A and B are Tarjan model constants while Du_e represents the apex diameter of the hydrocyclone e and Do_e represents the diameter of the vortex finder of the hydrocyclone e. The cut size can be estimated using Trawinski's (1976) expression as follows:

$$d50_e = 44 \cdot \left(\frac{Di_e \cdot Do_e}{Lc_e}\right)^{0.5} \cdot \left(\frac{\mu}{\rho_s - \rho_l}\right)^{0.5} \cdot Pf_e^{0.5} \qquad \forall e \in E_H \tag{13}$$

where Di_e is the inlet diameter of the hydrocyclone e, Lc_e is the height of hydrocyclone e, μ is the viscosity of the fluid, ρ_s is the density of the solid, ρ_l is the density of the liquid, and Pf_e represents the pressure of the inlet feed of the hydrocyclone e. It is known that the size of the hydrocyclones used in metallurgical operations can be determined from experimental results (Mular and Jull, 1980). The diameter of the vortex finder, Do_e, of the hydrocyclone should be approximately 35–40% of the diameter of the hydrocyclone, Dc_e, while the apex diameter should be approximately 15–20% of Dc_e. Keeping these facts in mind, it was considered that $Di_e = Dc_e \cdot 0.14$; $Do_e = Dc_e \cdot 0.40$; $Du_e = Dc_e \cdot 0.20$; and $Lc_e = Dc_e \cdot 3$, for $\forall e \in E_H$. However, other models can also be used (for more models, see Kraipech et al., 2006).

It is necessary to specify the capacity of each hydrocyclone. The capacity can be calculated using the following expression (Arterburn, 1982):

$$Q_e = 0.000199 \cdot Dc_e^{1.87} \cdot Pf_e^{0.5} \qquad \forall e \in E_H \tag{14}$$

Eq. (14) allows one to calculate the capacity of a hydrocyclone when its diameter, Dc_e, and feed inlet pressure, Pf_e, are known.

To estimate the cost of a hydrocyclone, Ch_e, Ruhmer's expression (1991) can be used. The expression relates the cost of a hydrocyclone to its diameter, Dc_e, in the following manner:

$$Ch_e = a_h + b_h \cdot Dc_e + c_h \cdot Dc_e^2 \qquad \forall e \in E_H \tag{15}$$

where a_h, b_h, and c_h are constants. For smaller diameters, Eq. (15) can be approximated by one or more linear equations as follows:

$$Ch_e = a_h \cdot y_h + b_h \cdot Dc_e \qquad Dc_e^{LO} \cdot y_h \leqslant Dc_e \leqslant Dc_e^{UP} \cdot y_h \qquad \forall e \in E_H \tag{16}$$

Eq. (16) is a linear equation bounded by the smaller diameters considered in this study. In the equation, a_h represents the fixed cost of the hydrocyclone and b_h represents its variable cost. Dc_e is the diameter of the hydrocyclone, with Dc_e^{LO} and Dc_e^{UP} being the lower and upper bounds of the diameter. y_h is a binary variable that represents the existence or nonexistence of the hydrocyclone e.

2.3. Thickening system superstructure

Equations similar to Eqs. (1)–(6) must be satisfied in case of the thickening system as well:

$$\sum_{s \in S_{T_{e,s}}^{in}} W_{s,c}^{E_T} - \sum_{s \in S_{T_{e,s}}^{out}} W_{s,c}^{E_T} = 0 \qquad \forall c \in C, \quad e \in E_T \tag{17}$$

Eq. (17) is a mass balance equation for the different pieces of equipment. $W_{s,c}^{E_T}$ is the mass flow rate for the particles of different sizes and the pulp volumetric flow rate in the thickening system superstructure, $S_{T_{e,s}}^{in} = \{(e,s)/s$ is an input stream $s \in S_T$ to the equipment $e \in E_T\}$, $S_{T_{e,s}}^{out} = \{(e,s)/s$ is an output stream $s \in S_T$ to the equipment $e \in E_T\}$.

$$\bigvee_{j \in J_T} \begin{bmatrix} y_{T_{e,j}} \\ W_{s_1,c}^{E_T} \geqslant n_{T_j} \cdot W_{s,c}^{E_T} \\ W_{s_2,c}^{E_T} \geqslant (1-n_{T_j}) \cdot W_{s,c}^{E_T} \end{bmatrix} \quad \forall e \in E_T, \quad c \in C, \quad s, s_1, s_2 \in S_T \tag{18}$$

Eq. (18) is analogous to Eq. (2) and discretises the decision of splitting the flow into a set of J_T alternatives, each of which is defined by the parameter n_{Tj}. In Eq. (18), $y_{T_{e,j}}$ is a binary variable representing the various disjunctions available and s is the stream whose flow is divided into streams s_1 and s_2. The parameter n_{Tj} is the fraction of the stream s that is sent to s_1. Eq. (18) can be written using the big M method with equations similar to Eqs. (4)–(6); for the sake of simplicity, these equations are not shown here.

A simple model of the thickener was proposed by King (2001); this model was for a continuous cylindrical thickener based on the ideal Kynch model and the sedimentation velocity of solids as determined by Richardson and Zaki (1954).

At steady state, the mass flow of particles with size t in the thickener underflow, $W_{s_u,t}^{E_T}$, is equal to the feed mass flow rate in the thickener feed, $W_{s,t}^{E_T}$. That is to say,

$$W_{s_u,t}^{E_T} = W_{s,t}^{E_T} \qquad \forall (s, s_u) \in S_e^E, \quad t \in T \tag{19}$$

where $S_e^E = \{(s, s_u)/s, s_u \in S_e^E, s_u$ is the underflow stream of the thickener $e \in E_T$ that is fed by stream $s\}$. The solid flux through any horizontal plane, f_e, in a steady-state thickener must be equal to the feed flux. Then, at the feed,

$$f_e = \sum_t W_{s,t}^{E_T}/\rho_s Ae_e \qquad \forall e \in E_T \tag{20}$$

And at the underflow discharge,

$$f_e = W_{s_u,p}^{E_T} C_{D,e}/Ae_e \qquad \forall e \in E_T \tag{21}$$

where Ae_e is the area of the thickener e, $W_{s_u,p}^{E_T}$ is the pulp volumetric flow rate in the underflow stream, and $C_{D,e}$ is the volumetric solid concentration in the discharge. $C_{D,e}$ can be calculated using the following equations (King, 2001):

$$C_{M,e} = C_{D,e}(f_e - \psi_{M,e})/f_e \qquad \forall e \in E_T \tag{22}$$

$$C_{M,e} = (\psi_{M,e} - f_e)/\psi_{M,s}^1 \qquad \forall e \in E_T \tag{23}$$

In Eq. (23), $C_{M,e}$ is an intermediate concentration and $\psi_{M,e}$ is the setting flux at $C_{M,e}$, which can be calculated using the Richardson–Zaki model for sedimentation. Thus,

$$\psi_{M,e} = v_{TF} C_{M,e}(1 - r_F C_{M,e})^n \qquad \forall e \in E_T \tag{24}$$

$$\psi_{M,e}^1 = v_{TF}(1 - r_F C_{M,e})^n - n v_{TF} r_F C_{M,e}(1 - r_F C_{M,e})^{n-1} \qquad \forall e \in E_T \tag{25}$$

where v_{TF}, r_F, and n are constants. Eqs. (22)–(25) are valid if the feed rate is less than the maximum possible feed rate of solids into the thickener. Thus,

$$\sum_t W_{s,t}^{E_T} \leqslant \rho_s f_{e,\max} Ae_e \qquad \forall e \in E_T \tag{26}$$

$$f_{e,\max} = \frac{4 v_{TF}}{r_F} \frac{n(n-1)^{n-1}}{(n+1)^{n+1}} \tag{27}$$

The model used for the thickener is based on assumptions that are not valid in all situations. For instance, they are not valid for overflow steady-state situations. More complete formulas can be obtained from the literature (see Diehl, 2001, for steady-state Kynch theory, and Bürger and Marváez, 2007, regarding the steady state in flocculated suspensions); however, in such cases, the solution of the optimization problem can be more complex.

To estimate the cost of the thickener, Ct_e, the expression given by Parkinson and Mular (1972) can be used; in the expression, the cost is related to the thickener diameter, Dt_e.

$$Ct_e = a_t \cdot Dt_e^{b_t} \quad \forall e \in E_T \tag{28}$$

where a_t and b_t are constants. Eq. (28) can be approximated by one or more linear equations for smaller diameters as follows:

$$Ct_e = a_t \cdot y_t + b_t \cdot Dt_e \qquad Dt_e^{LO} \cdot y_t \leqslant Dt_e \leqslant Dt_e^{UP} \cdot y_t \qquad \forall e \in E_T \tag{29}$$

Eq. (29) is a linear equation bounded by the smaller diameters used in this study. In the equation, a_t represents the fixed cost of the thickener and b_t represents its variable cost. Dt_e^{LO} and Dt_e^{UP} are the lower and upper bounds for the thickener diameter, and y_t is a binary variable that represents the existence or nonexistence of the thickener e.

2.4. Stream connections

With the goal of interconnecting the OS with the hydrocyclone and thickener superstructures, certain streams must be equal. For example, the bottom stream exiting the feed divider (in the overall superstructure, Fig. 1) that goes towards the hydrocyclone system is also the feed stream in the hydrocyclone system (Fig. 2). In addition, the overflow stream in the thickener system superstructure (Fig. 3) is the same as the stream exiting the thickener system that goes to the clear water mixer in the overall superstructure. These equalities can be written as follows:

$$W_{s,c} = W_{s_H,c}^{E_H} \qquad \forall (s, s_H) \in AH_{s,s_H} \tag{30}$$

$$W_{s,c} = W_{s_O,c}^{E_T} \qquad \forall (s, s_O) \in OT_{s,s_O} \tag{31}$$

where $AH_{s,s_H} = \{(s, s_H)$ / stream s in the OS is also the stream s_H in the hydrocyclone superstructure$\}$, and $OT_{s,s_O} = \{(s, s_O)$ / stream s in the OS is also the stream s_O in the thickeners superstructure$\}$.

In addition to these restrictions, it is essential that the feed flow to the overall superstructure is known.

2.5. Optimization problems

The selection of an optimal structure depends on the desired objectives. In this study, two objective functions were identified. The first objective was the determination of the maximum water recovery rate. That is to say,

$$\text{Maximize} \qquad R = W_{s,p} \qquad \forall s \in S, \quad p \in P \tag{32}$$

Subject to the constraints : (1) to (14) and (17) to (27)

In Eq. (32), R is the water recovery. For this maximization problem (P1), the variables are the flows in each of the process streams in all the superstructures and the binary variables used to represent the disjunction decisions. The parameters used in this problem are the equipment design data, that is, the diameters of the various pieces of equipment and the pulp properties.

The second objective was to minimize the equipment cost, namely the cost associated with the hydrocyclone and thickeners systems. That is to say,

$$\text{Minimize} \quad Z = \sum_{e \in E_H} (Ch_e) + \sum_{e \in E_T} (Ct_e) \quad \forall e \in E_H \cup E_T \tag{33}$$

Subject to the constraints :(1) to (31)

In Eq. (33), Z is the objective function to be minimized, which, in this case, is the total cost of the equipment. This is the cost associated with the hydrocyclone as per Eq. (16) and the cost associated with the thickener as per Eq. (29). For the cost minimization problem (P2), the variables used correspond to the flows in each of the process streams in each of the superstructures involved, as well as the diameters of the hydrocyclones and thickeners and the binary variables representing the selection of the various pieces of equipment. The parameters used in this problem are fixed and are the variable equipment costs, the desired mass flow in the clear water stream, and the pulp properties.

3. Case studies

In this section, we discuss the application of the method proposed in Section 2 for the recovery of water from tailings. The method was applied in three cases. First, the method was validated by comparing it with that used in an actual industrial plant; second, the method was applied to process design, both for the maximum water recovery problem (P1) and the minimum equipment costs problem (P2), for a dewatering system having one hydrocyclone and one thickener; and finally, the method was applied to the process design (P1) of systems with more than one thickener and hydrocyclone. In all the cases, the particles were classified as being 300, 210, 150, 105, 75, 53, 45, 38, and 19 m in size. The main data used were common to all case studies and are presented in Table 1.

All the cases studies were solved using Branch-And-Reduce Optimization Navigator (BARON), which is a computational system

Table 1
Data used in all the case studies.

Parameter	Value
Water density, (t/m^3)	1.00
Solid density, (t/m^3)	2.70
Water viscosity, (Cp)	2.50
Sedimentation rate, (m/h)	1.09
Hydrocyclone feed pressure, (kPa)	69

for solving non-convex optimization problems to global optimality. BARON was run under the General Algebraic Modelling System (GAMS) modelling language on a computer system with an Intel Xeon (3.3 GHz) processor.

3.1. Industrial application (P1)

In the first case study, the P1 model was applied to an industrial plant in order to validate the proposed method. The industrial plant is a copper concentrator plant located in northern Chile. This industrial plant has a circuit for the recovery of water from flotation tails, and this circuit consists of a hydrocyclone and a thickener. The P1 model was studied experimentally for different structures and different mass flow rates of the feed stream between the hydrocyclone and the thickener. The circuit was fed by a slurry at 19,100 m^3/h; the slurry contained 24% solids. The solids were classified into 9 sizes. An optimal structure was identified on the basis of the experimental results. This structure is shown in Fig. 4; 10% of the feed was sent to the hydrocyclone (Huerta, 2008).

The plant data were used to tune the thickener and hydrocyclone models to describe the solid mass flow rate for each particle size. It was found that results obtained using the model and the plant data were in good agreement (see Fig. 5).

Then, model P1 was used to obtain the optimal structure, which was the same as the experimentally determined optimum circuit (Fig. 4). 15% of the feed flow was sent to the hydrocyclone and the remainder of the feed was sent to the thickener. This configuration achieved a water recovery of 68.4%. This result was in agreement with the experimental results obtained from the plant. Further details of the main process stream flows as well as other important parameters are listed in Table 2.

3.2. Second case study (P1 and P2)

In a second case study, a system consisting of a hydrocyclone and a thickener was used. The P1 and P2 models were used to

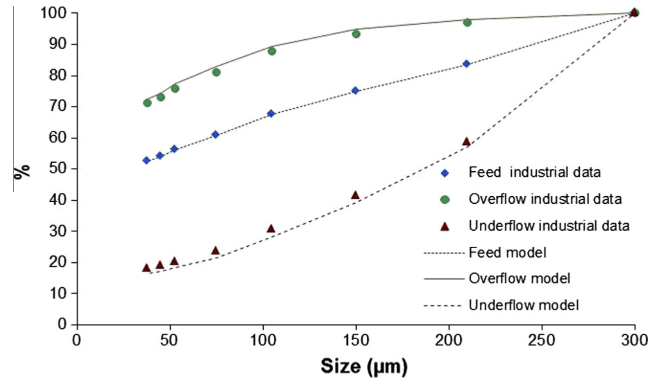

Fig. 5. Size distributions (experimental and calculated) for mass balance in the hydrocyclone battery.

determine the maximum water recovery rate and the minimum equipment cost, respectively, and to design the dewatering system structure (P1 and P2) and to determine the operational conditions (P1 and P2) and equipment sizes (P2). The circuit was fed by a slurry at 1200 m^3/h; the slurry contained 24% solids.

The maximum water recovery (P1) problem was solved using several sets of split flows (for both the feed slurry and the hydrocyclone overflow). Several cases (i.e., cases with 2, 3, and 6 alternatives) had unfeasible solutions for the restrictions placed on the equipment sizes and capabilities. The optimal solution corresponded to 846.82 t/h of water being recovered with the structure shown in Fig. 6; 3/4 of the feed was sent to the thickener and 1/3 to the hydrocyclone. With 5 alternatives for dividing the stream (from 0 to 1 in increments of 0.25), the problem had 178 continuous variables, 10 binary variables, and 334 constraints. The CPU time was 0.170 s. Further, with 21 alternatives for dividing the stream (from 0 to 1 in increments of 0.05), the problem had 210 continuous variables, 42 binary variables, and 910 constraints. The CPU time was 0.760 s. This indicates that the replacement of the bilinear expressions for mass balance by disjunctions of the streams that can be divided (Eqs. (2)–(6)) has an effect on the obtained solution and the time required to solve the problem.

Using the case with five alternatives for the stream divider, the thickener diameter was increased by 9.7%, resulting in a 9.1% increase in water recovery; however the optimal structure (Fig. 6) did not change, including the manner in which the stream was divided (3/4 of the feed slurry to the thickener and 1/3 to the hydrocyclone). Similarly, the diameter of the hydrocyclone was

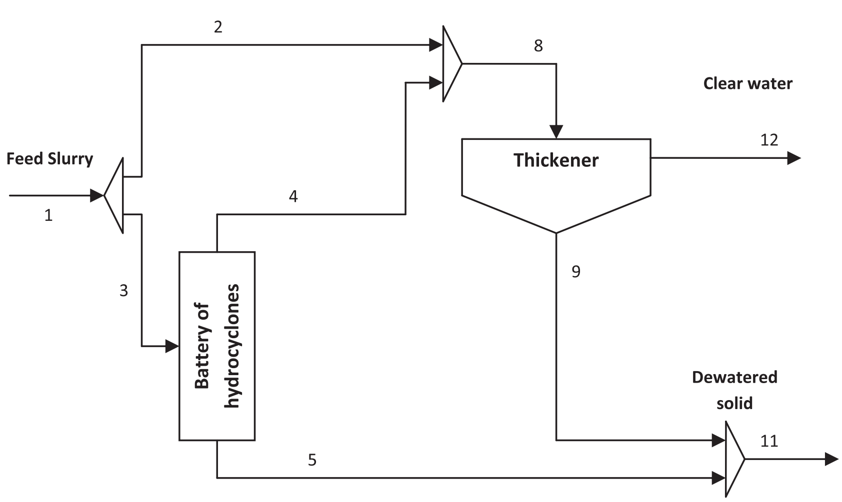

Fig. 4. Optimal solutions (experimental and theoretical) for case study 1.

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Table 2
Mass flow rates for the optimal circuit in Fig. 4.

Streams	1	3	4	5	9	11	12
Mass flow rate (t/h)	22,500	3375	2783	592	10,208	10,800	11,700
Water (t/h)	17,100	2565	2363	202	5198	5400	11,700
Solids (t/h)	5400	810	420	390	5010	5400	0
Solids (%)	24	24	15.1	65.9	49.1	50	0

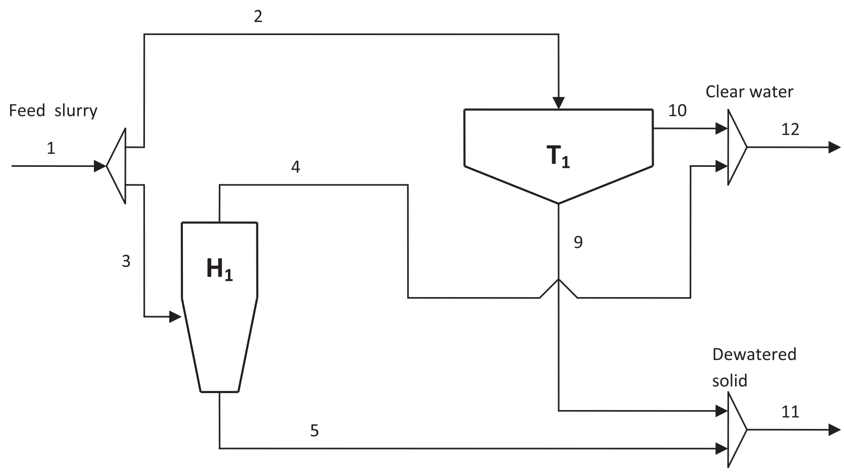

Fig. 6. Optimal solution for case study 2.

increased by 8%, 15%, and 23%; however, this resulted in negligible differences in the results obtained. Then, the number of alternatives was increased to 21 (from 0 to 1 in increments of 0.05), and new solutions were observed. For example, for a 23% increase in the hydrocyclone diameter, water recovery increased by 8.9%; this entailed sending 65% of the feed slurry to the thickener.

Fig. 6 shows the structure that was obtained by solving the problem of minimizing the system cost (P2). A minimum water recovery rate of 846 m^3/h was used as a constraint. The calculated design variables were the diameters of the spigot, vortex, and cyclone, the slurry feed head for the hydrocyclone, and the diameter of the thickener. The feed slurry and the thickener overflow were divided to recover water and reduce equipment costs. With 5 alternatives for stream division (from 0 to 1 in increments of 0.25), the problem had 202 continuous variables, 12 binary variables, and 512 constraints. The CPU time was 0.97 s. With 21 alternatives for stream division (from 0 to 1 in increments of 0.05), the problem had 234 continuous variables, 44 binary variables, and 1088 constraints. The CPU time was 2.20 s. The solutions obtained using 5 and 21 alternatives for stream division had the same structure; however, in the case of 5 alternatives, 75% of the feed slurry is sent to the thickener, while in the case of 21 alternates, 65% of the feed slurry is sent to the thickener. This meant that the equipment sizes and costs (343,065 and 310,422 USD) were different in the case of 5 and 21 alternatives, respectively.

Moreover, the solution for the case with five alternatives for stream division resulted in the lowest value allowed for the water recovery rate (846 t/h). On the other hand, with 21 alternatives, the water recovery was slightly higher than the minimum (by 0.2%).

If the minimum value of water recovery rate was increased by 12%, the costs increased by 24% in the case of 5 alternatives for stream division and by 15% in the case of 21 alternatives. The

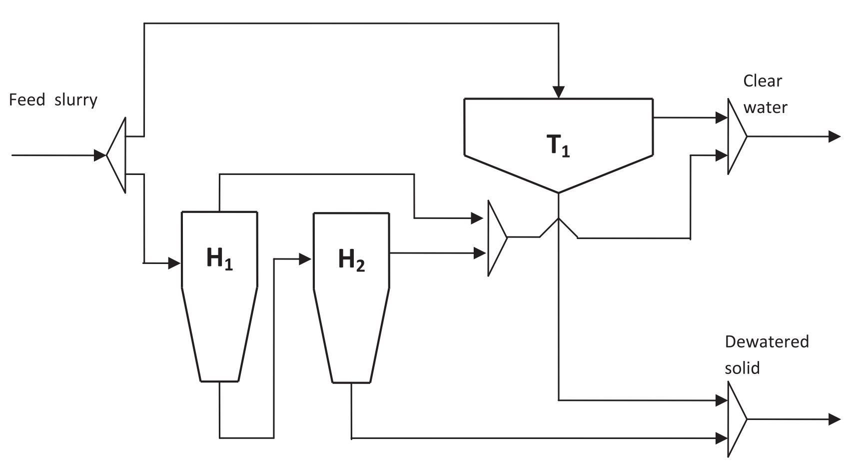

Fig. 7. Optimal solution for case study 3.

structures obtained were the same; the thickeners were larger, while the hydrocyclones were of the same size.

3.3. Third case study (P1)

In a third case study, a system consisting of two hydrocyclones and two thickeners was used. The P1 model was used to determine the maximum recovery of water, to design the dewatering system structure, and to determine the operational conditions, for given equipment sizes. The number of structure alternatives in the OS was 9 (two stream dividers). In addition, there were 81 structures in the hydrocyclone system superstructure (four stream dividers) and 9 structures in the thickener system superstructure (two stream dividers). Thus, the total number of structure alternatives from which to determine the optimal structure were 6561.

Fig. 7 shows the structure that was obtained by solving the maximization problem with 5 and 21 alternatives for stream division in all the divisors in all the superstructures. The feed slurry was divided to recover water.

With 5 alternatives for stream division (from 0 to 1 in increments of 0.25), the problem had 420 continuous variables, 40 binary variables, and 1069 constraints. The CPU time was 13.92 s. With 21 alternatives for stream division (from 0 to 1 in increments of 0.05), the problem had 548 continuous variables, 168 binary variables, and 3373 constraints. The CPU time was 68.69 s. The CPU time was significantly higher when compared to that for the previous case, owing to the combinatorial nature of the problem. Further, it can be expected that, in the case of problems with more pieces of equipment, the CPU time will increase exponentially.

4. Conclusion and comments

A method for designing an optimal water recovery separation circuit was presented. Two problems were solved: the first problem optimized water recovery from slurry for specified equipment sizes and the second problem was concerned with determining the minimum cost of a dewatering system for a specified rate of water recovery.

The method was validated by applying it to an actual industrial plant; the model predictions and the experimental results were found to be in good agreement. Several case studies were performed, and the results indicated that the proposed method can be useful for designing new dewatering systems or improving existing ones. However, more research is needed to determine the effect of the quality of the hydrocyclone and thickener models and stream splitters.

This is because simple models were used for the hydrocyclones and thickeners. While these can be replaced by more complete ones, this may result in complex optimization problems, and it may be difficult to reach convergence and/or a local optimum. Hence, we recommend using simple models for the hydrocyclone and thickeners, like those used in this work, in order to determine the optimal system structure. Then, once the optimal structure is known, the equipment can be designed and optimized using more complex models obtained with the help of tools such as computational fluid dynamics (CFD) simulations.

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