Two-Dimensional Energy Balance Model and Its Application to Some Climatic Issues

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ABSTRACT

Based on a two-dimensional energy balance model, the studies on some climatic issues such as the relationship between ice cap latitude and solar constant, desertification, and the warming effect of carbon dioxide, have been reviewed and discussed. The phenomenon that a fixed solar constant might correspond to different equilibrium ice cap latitudes is determined by the continuity of albedo distribution. The discontinuity in albedo distribution increases the number of equilibrium ice cap latitudes. Desert would expand both northward and southward when desert surface albedo is increasing. This would deteriorate the ecological environment in border regions, and then threaten the existence of local inhabitants. Melting of the polar ice would not be accelerated, with increasing carbon dioxide concentration. The ice cap latitude would move northward slowly, with some "hiatus" periods, under the slowly increasing global average surface temperature. According to the current research, future development of the two-dimensional energy balance model and possible progress are also forecasted.

Key words: two-dimensional energy balance model, multiple equilibria, ice cap latitude, desertification, warming effect of carbon dioxide

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1. Introduction

Global change is one of the most active and fastest growing fields in the scientific research, and climate change is one of the major research priorities in global change (Ye et al., 2002). Changes in the radiation balance of the earth-atmospheric system caused by greenhouse gases and aerosol concentrations will alter the energy balance of the climate system and cause global climate change (Qin et al., 2007). The negative impacts of climate change on natural ecosystems, economy, and national security system gain far more attention than its positive impacts (Qin, 2004). A series of serious environmental issues accompanied with climate change has threatened the development of human society. It has become an urgent need for the human survival and development to avoid possible environmental disasters and to achieve the sustainable economic and social development through scientific and rational way (orderly human activities) (Ye et al., 2001).

Mankind has not yet come to really understand the driving forces of climate change on different timescales and the complex response–feedback processes in the climate system, even though the contribution of increasing greenhouse gases to global warming has already been recognized (Liu, 2006). For one example, ice core data analysis showed that there is no direct causal relationship between greenhouse gases concentration and atmospheric temperature and temperature change could even lead the concentration change of greenhouse gases (Cui et al., 2012). For another example, the linear trend of global average temperature during 1999–2008 is close to zero, significantly lower than previous projection (Knight et al., 2009); however, concentration of greenhouse gases has

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steadily increased. Therefore, it not only has important theoretical significance but also provides some practical guidance to the orderly human activities to deeply analyze and understand the various driving factors and feedback processes which contribute to the global climate change.

Climate and earth system models are powerful research tools which could be used to quantitatively analyze the impacts of certain driving factors or feedback processes on global or even regional climate. This could no doubt deepen the understanding of climate change, project for future change, and provide reference for decision makers. In addition, some simple theoretical models with solid physical basis could also be used to study climate change. These models could highlight the driving factors for climate change, analyze in-depth the physical processes, supplement other models' results theoretically and physically, and improve and enrich the climate change research. The simplest approach is to consider the earth system as a particle satisfying the energy conservation, which could be used to solve the average temperature of the system. This forms the zero-dimensional energy balance model (EBM). It is easy to introduce the greenhouse effects and ice albedo feedback mechanism into the zero-dimensional EBM through revising some relative parameters (Budyko, 1969; North et al., 1981).

One-dimensional EBM could be derived easily by introducing the latitudinal temperature distribution into the zero-dimensional EBM. The classical studies by Budyko (1969), Sellers (1969), and North (1975a) are most representative and have had a prolonged impact on ensuing research. Two-dimensional EBMs (Sellers, 1976; Chao and Chen, 1979) could also be constructed by further introducing longitudinal or vertical physical processes. The role of oceans could also be considered (Shi et al., 1996). Extending the zero-dimensional EBM vertically gets one-dimensional radiative-convective model (Manabe and Strickler, 1964; Manabe and Wetherald, 1967) which could not be solved through analytical methods. To obtain the analytical solution, it is necessary to simplify the long-wave radiation absorption spectrum. For example, Chao and Chen (1979) developed a two-dimensional EBM by using the simplified longwave radiation absorption scheme introduced by Kuo (1973). Based on this two-dimensional EBM, they studied the relationship between ice cap latitude and solar constant (Chao and Chen, 1979), desertification (Chao and Li, 2010a), and warming effects of carbon dioxide (Chao and Li, 2010b). Their studies would be introduced, improved, and developed in the following sections.

2. Two-dimensional EBM

The average atmospheric temperature field is controlled by many factors, such as the latitudinal distribution of solar radiation, radiation propagation and conversion processes, turbulent heat exchange, latent heat caused by condensation and evaporation processes, and the heat exchange caused by cold and warm advection. The temperature equation could be written as (Ye and Zhu, 1958)

$$\rho_{\mathrm{a}} C_p \frac{\mathrm{d}T}{\mathrm{d}t} - \frac{\mathrm{d}p}{\mathrm{d}t} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \tag{1}$$

where ρ_{a} is air density, C_p is specific heat of air, T is air temperature, and p is the pressure; ε_1, ε_2, and ε_3 are the heat flux caused by radiation, turbulence, and condensation or evaporation, respectively.

Set k_j as the longwave radiation absorption coefficient at the wavelength λ_j; k' as the mean radiation absorption coefficient for solar radiation; $A_j(B_j)$ as the downward (upward) longwave radiation during the wavelength interval $\Delta\lambda_i$; E_j as the black body radiation during the same wavelength interval $\Delta\lambda_i$; Q as the downward solar radiation. Then, the heat flux caused by radiation could be written as

$$\varepsilon_1 = \sum_j k_j (A_j + B_j - 2E_j) + k'Q. \tag{2}$$

According to Bouguer-Lambert law

$$\frac{\partial A_j}{\partial z} = k_j (A_j - E_j), \tag{3}$$

$$\frac{\partial B_j}{\partial z} = k_j (E_j - B_j), \tag{4}$$

$$\frac{\partial Q}{\partial z} = k'Q. \tag{5}$$

The heat flux caused by turbulence could be expressed as

$$\varepsilon_2 = \rho_a C_p K_h \nabla_h^2 T + \rho_a C_p K \frac{\partial^2 T}{\partial z^2}, \quad (6)$$

where K_h is horizontal turbulent thermal diffusivity ($\kappa_h = \rho_a C_p K_h$ is called horizontal turbulent thermal conductivity); K is vertical turbulent thermal diffusivity ($\kappa_z = \rho_a C_p K$ is called vertical turbulent thermal conductivity); ∇_h^2 is the horizontal Laplacian operator. Note that K_h and K are set to constants. Temperature Eq. (1) could be simplified by the assumption that the radiation transfer processes are balanced by turbulent heat flux. The zonal mean form of Eq. (1) under spherical coordinates could be written as

$$\frac{\kappa_h}{a^2}\frac{\partial}{\partial x}(1-x^2)\frac{\partial T}{\partial x} + \frac{\partial}{\partial z}\Big(\kappa_z \frac{\partial T}{\partial z}\Big) + \sum_j k_j (A_j + B_j - 2E_j) + k' Q = 0. \quad (7)$$

where $x = \sin\phi$, ϕ is latitude, and a is the radius of the earth. To solve the above equation, integral along the entire radiation absorption spectrum is needed, which is complex and inconvenient for theoretical analysis that copes with the radiation transfer processes analytically or semi-analytically. Kibel (1943) had established strict temperature distribution theory in the early 1940s, and Blinova (1947) further developed and refined the theory. They considered the entire radiation absorption spectrum as a whole to avoid complicated integral calculation and eventually derived a fourth-order partial differential temperature equation by assuming radiation absorption coefficients as constants. Detailed introduction could be found in Ye and Zhu (1958). Kuo (1973) further divided the entire longwave radiation absorption spectrum into two parts (strong and weak absorptive zones, presented by subscripts s and w) by using the criterion $k_j \gtrless \mathrm{d}/\mathrm{d}z$. The mean radiation absorption coefficients are defined as

$$k_{s,w} = \frac{\int_{s,w} k_j E_j \mathrm{d}\lambda}{E_{s,w}}. \quad (8)$$

Meanwhile, we define $E_s = rE$, $E_w = (1-r)E$, and $E = \sigma T^4$.

Introduce optical depth

$$\xi = \frac{1}{\xi_0}\int_z^\infty k' \mathrm{d}z, \quad \xi_0 = \int_0^\infty k' \mathrm{d}z, \quad (9)$$

according to the simplified scheme, it could be derived from Eq. (7) that

$$D\frac{\partial}{\partial x}(1-x^2)\frac{\partial E}{\partial x} + (\kappa_z + \kappa_r)\frac{\partial^2 E}{\partial \xi^2} - N^2 E = -\widetilde{S}\overline{Q}_0 \xi_0 \mathrm{e}^{-\xi_0 \xi} S(x) + \widetilde{S}\overline{Q}_0 C, \quad (10)$$

where

$$D = \frac{\xi_0^2 \kappa_h}{k'^2 a^2}, \quad \widetilde{S} = \frac{4\xi_0 \sigma \overline{T}^3}{k'}, \quad \kappa_r = \frac{8r\sigma \overline{T}^3}{k_s},$$
$$N^2 = \frac{8(1-r)k_w \xi_0^2 \sigma \overline{T}^3}{k'^2},$$

and $\overline{Q}_0$ is the mean solar radiation arriving at the upper bound of atmosphere. Compared with κ_z, κ_r can be called the equivalent radiation exchange coefficient, which represents the radiative effect in the strong absorptive zone. N^2 is called Newtonian radiative cooling coefficient, which represents the radiative effect in the weak absorptive zone. Brunt (1934) pointed out that the role of the strong absorptive zone is much like thermal conduction process and the role of weak absorptive zone could take the form of Newtonian cooling (quoted from Chen and Chao, 1988). Kuo (1973) unified these two effects into one simple equation. However, he left the specific form of the integral constant C unsolved. Chao and Chen (1979) derived the integral constant C by integral over the globe, which could be written as

$$C = r\Big(1 + \frac{k'}{k_w}\Big)\overline{\prod}_0 + (\overline{\prod}_1 - \overline{\prod}_0 + 1 - \mathrm{e}^{-\xi_0}). \quad (11)$$

The upper and lower boundary conditions are taken as

$$\xi = 0, \quad \int_0^1 E_0 \mathrm{d}x = \frac{1}{2}\Big(1 + \frac{k'}{k_w}\Big)\overline{\prod}_0 \overline{Q}_0, \quad (12)$$

$$\xi = 1, \quad (\kappa_z + \kappa_r)\frac{\partial E}{\partial \xi} = N^2 \int_0^1 E \mathrm{d}\xi - 2(1-r)\widetilde{S}E_0 + \widetilde{S}\overline{Q}_0\Big(\frac{k'}{k_w}\overline{\prod}_0 + \overline{\prod}_1\Big)S(x), \quad (13)$$

where $\overline{\prod}_0 = 1 - \Gamma_p$ and $\overline{\prod}_1 = 1 - \Gamma_p - (1 - \mathrm{e}^{-\xi_0})$ are net solar radiation proportion arriving at the upper bound of atmosphere and at the ground, respectively. $\overline{\prod}_0 = \int_0^1 \prod_0 S(x)\mathrm{d}x$ and $\overline{\prod}_1 = \int_0^1 \prod_1 S(x)\mathrm{d}x$ are their global integral values. Chao and Chen (1979) took $\prod_0 = 1$, which means that the planetary albedo at the upper boundary is neglected.

Use Legendre series to solve Eq. (10). Set

$$E(x,\xi) = \sum_n E^{(n)}(\xi) P_n(x), \tag{14}$$

extend relevant variables into Legendre series and solve the coefficients $E^{(n)}(\xi)$, and then the analytical solution could be obtained. This two-dimensional EBM is then called the Kuo-Chao-Chen model (KCCM). Chen (1982a, b) further analyzed the stability of the solution and the sensibility of the parameters. The above KCCM has a wide spread application in the climate change field, such as ice cap sensitivity against solar constant (Chao and Chen, 1979), planetary atmospheric temperature distribution (Lü and Chao, 1981), desertification (Chao and Li, 2010a), and warming effects of carbon dioxide (Chao and Li, 2010b).

3. Relationship between ice cap latitude and solar constant

Many scientists studied the reasons of the cold period in the 1960s. The studies of Budyko (1969), Sellers (1969), and North (1975a) are most representative. Ice cap latitude is very sensible against solar constant in one-dimensional EBM (Fig. 1). Only a 2% decrease in solar constant would push the ice cap to move southward till around 50°N (glacial climate), even around the equator (a snowball earth). On the other hand, a certain solar constant (for example, the current value) might correspond to multiple ice cap latitudes, which means multiple climate states. These two phenomena could be interpreted into two questions. One is the sensitivity of the ice cap against solar constant and the other is the stability and physical meanings of different equilibrium climate states.

Lindzen and Farrell (1977) pointed out that the dependence of the ice cap on solar constant could be reduced just by adding some more realistic heat transport processes on the one-dimensional EBM. Lian and Cess (1977) found that the dependence could be reduced by correcting the relationship between temperature and albedo. Making a more reasonable revision on the parameterization between longwave radiation and temperature through satellite data, Oerlemans and Van Den Dool (1978) suggested that glacial climate occurs only when solar constant reduces its value by about 9%. Generally speaking, due to lack of vertical energy rearrangement, one-dimensional EMBs are more sensible to solar constant variation. Chao and Chen (1979) re-examined the issue with KCCM and noted that about a 15% decrease in solar constant could push the current climate into a glacial one, which means that two-dimensional EBM could greatly reduce the sensibility.

Budyko (1972) firstly discovered the multiple equilibria characteristic in the simple one-dimensional EBMs. He pointed out that a stable equilibrium point satisfies $\mathrm{d}Q/\mathrm{d}x_s > 0$ and an unstable one corresponds to $\mathrm{d}Q/\mathrm{d}x_s < 0$. Chýlek and Coakley (1975) subsequently analyzed the multiple equilibria phenomenon in ice cap solution. North (1975a, b) found two equilibrium ice cap latitudes, which were south near the polar region and close to each other. He thought that the unstable equilibrium state closer to the polar point a little unreasonable and removed it by adjusting the zonal distribution of solar radiation. Held and Suarez (1974) analyzed the albedo feedback mechanism in one simple EBM and his conclusion supported the results of Budyko (1972). They contributed the occurrence of the two equilibria near the polar region to the introduction of the temperature-related diffusive term. When diffusion was small enough, there was no significant physical discrepancy between the two equilibria. Ghil (1976) found there were three equilibria corresponding to inter-glacial, glacial, and snowball earth climate, respectively. Drazin and Griffel (1977) further analyzed the characteristic of the multiple equilibrium solutions and indicated that one equilibrium solution would be unstable if its corresponding eigenvalues were smaller than zero. Lin (1978) removed the unstable equilibrium closer to the polar region by prescribing the nonlinear turbulent exchange coefficients. Cahalan and North (1979) suggested that the unstable equilibrium solution was caused by the step function of the albedo distribution and any form of smoothing of the albedo near the ice cap latitude could remove the unstable equilibrium solution, which is also mentioned by Coakley (1979).

The unstable equilibrium means that ice cap moves southward when solar constant increases its value. It seems unreasonable. Therefore, in the review paper about EBMs, North et al. (1981) disqualified the realistic existence possibility of the unstable equilibrium solution. They contributed it to the artificial step albedo function introduced for the mathematical facilitation. However, the results of Held and Suarez (1974) and Ghil (1976) seemed to have no such meanings. In fact, actual albedo distribution could experience large discontinuity, such as the great albedo discrepancy between Antarctic ice sheet and the surrounding sea water, in which situation the step function form of albedo seems more reasonable. Therefore, the unstable equilibrium might characterize one climate evolution under certain circumstance. It is inappropriate to consider the unstable equilibrium as a solution with no physical meaning and then directly remove it.

Chao and Chen (1979) contributed the ice cap latitude sensitivity to solar constant to ignoring the vertical heat transport in one-dimensional EBMs. They built the KCCM which could cope with the vertical energy transport to study the question further. According to their analysis, the temperature is insensitive to the solar constant. A glacial climate needs 15%–20% decrease of the solar constant. Meanwhile, there would be no significant difference in solutions when other parameters vary 20% of their values (Chen, 1982b). Like the one-dimensional model, albedo distribution in the two-dimensional model is taken as

$$\Gamma(x, x_s) = \begin{cases} \alpha_1(x), & x > x_s \\ \alpha_2(x), & x < x_s \end{cases}, \tag{15}$$

where $x_s = \sin\phi_s$, ϕ_s is the ice cap latitude and is determined by $T = -10$°C; α_1 and α_2 are the albedo distributions of the ice and non-ice surface, respectively, i.e., they could take continuous (North and Coakley, 1979), semi-continuous (Coakley, 1979), or discontinuous (Budyko, 1969) forms such as $\alpha_1(x) = \alpha_2(x) = a_0 + a_2P_2(x)$; $\alpha_1(x) = 0.62$, $\alpha_2(x) = a_0 + a_2P_2(x)$; and $\alpha_1(x) = 0.62$, $\alpha_2(x) = 0.132$, respectively. The distribution of the ice cap latitude in each case is shown in Fig. 2.

In the continuous albedo distribution case (solid line), the relationship between the ice cap latitude and the solar constant is monotonous. Polar ice sheet would shrink northward until emergence of an ice-free earth, with increasing solar constant. An ice-free earth needs a 10% increase of the solar constant and a glacial climate (50°N) needs a 20% decrease of the solar constant. In the semi-continuous case (dashed line), the monotonous relationship is destroyed. A critical ice cap latitude (near 30°N) divides the rela-

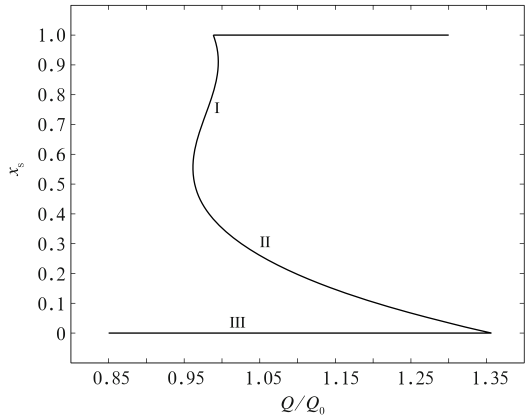

Fig. 1. The ice cap latitude x_s (sine of latitude) as a function of the solar constant in unit of the present solar constant (Q_0). [From North, 1975a]

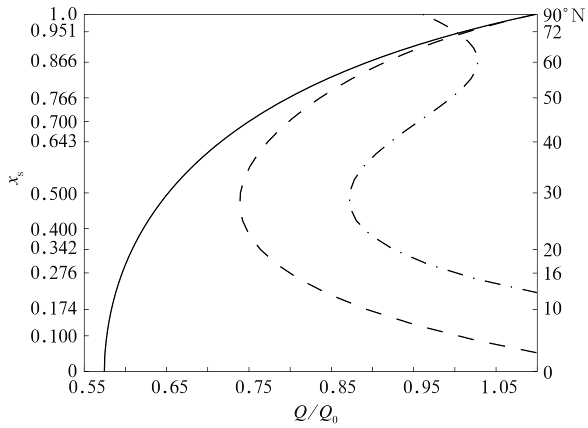

Fig. 2. The ice cap latitude as a function of the solar constant in unit of the present solar constant. The right and left y-axis coordinates are latitude and its sine, respectively. The solid, dashed, and dash-dotted lines mean that the albedo distribution is taken as $\alpha_1(x) = \alpha_2(x) = a_0 + a_2P_2(x)$; $\alpha_1(x) = 0.62$, $\alpha_2(x) = a_0 + a_2P_2(x)$; and $\alpha_1(x) = 0.62$, $\alpha_2(x) = 0.132$.

tionship into two parts. The upper part remains the same with the continuous case and the lower part suggests that bigger solar constant corresponds to larger polar ice range. The solar constant has to increase 10% to heat an ice-free earth and it has to decrease 16% to cool the earth into glaciation. In this case, one solar constant might correspond to two ice cap latitudes.

In the discontinuous case (dash-dotted line), the situation is similar to the semi-continuous case but with a more complex variation. There are two critical ice cap latitudes (near 30° and 60°N), which divide the relationship into three parts. The relationship for the 30°–60°N part means that bigger solar constant would melt more polar ice while for the other two parts, bigger solar constant accompanies with larger polar ice coverage. A given solar constant (such as the current value) could correspond to three equilibrium ice cap latitudes (near 72°, 50°, and 16°N), which seem to have occurred in the earth's evolutionary history. The Arctic ice sheet is around 72°N in the current climate; it marched to the midlatitude in the Carboniferous-Permian glaciation and Quaternary glaciation; and it might have arrived in low latitude, even the equator (a snowball earth), in the new Proterozoic glaciations (Sumner et al., 1987; Schmidt et al., 1991; Schmidt and Williams, 1995; Sohl et al., 1999).

To conclude, the relationship between ice cap latitude and solar constant is determined by the distribution of surface albedo. The relationship would be monotonous if given a continuous albedo distribution. That is, bigger (smaller) solar constant corresponds to smaller (larger) polar ice range. This monotonous characteristic would be destroyed by a discontinuous albedo distribution. One given solar constant might be linked with two even three ice cap latitudes. Therefore, the circumstance that bigger (smaller) solar constant could correspond to larger (smaller) polar ice range could occur. Although such a circumstance might be not easily understood and unstable, it is still retained for discussion. More detailed global surface albedo products derived from satellite data would contribute to a better understanding of this issue.

The theoretical calculation reproduces the surface temperature distribution well except for the higher temperature in the tropical region (Fig. 3a). It might

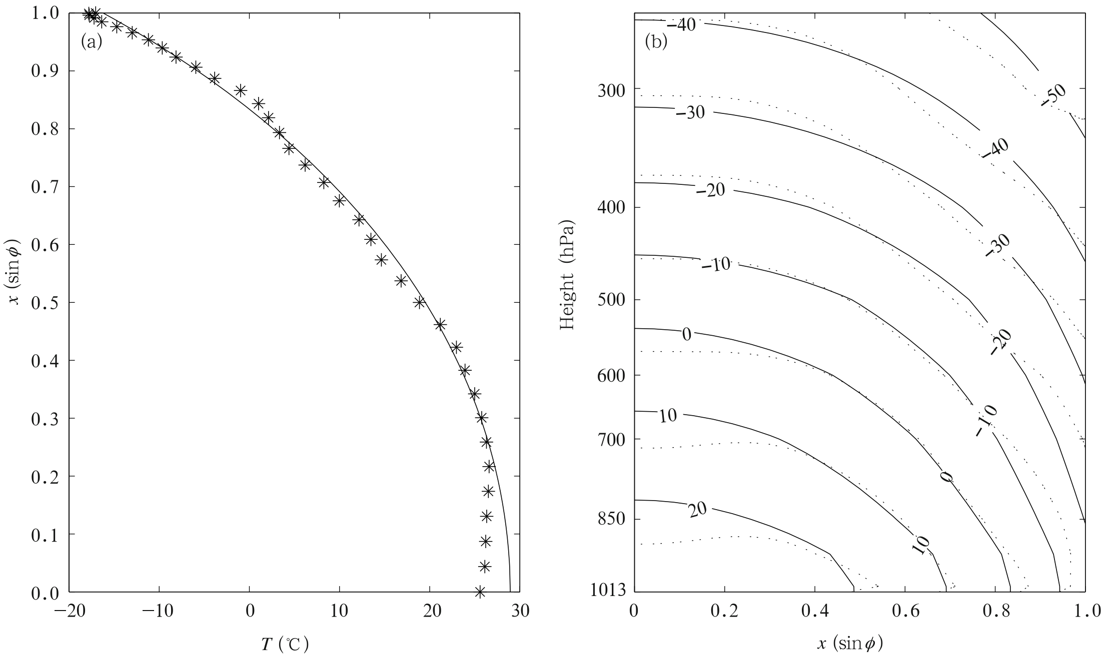

Fig. 3. Distributions of (a) surface temperature and (b) vertical temperature. Solid lines represent the theoretical results and the asterisk in (a) and the dotted lines in (b) are the temperature distribution derived from observation. The x-axis in (a) is temperature and (b) the sine of latitude. The y-axis in (a) is the sine of latitude and (b) vertical levels.

be caused by not reasonably reflecting the tropical heat transport by Hadley cell. If more realistic heat transport is introduced (Lindzen and Farrell, 1977), the temperature simulation in the tropics could be improved. Significant discrepancy between the theoretical and observed vertical temperature distributions could be resulted if the definition of optical depth using Eq. (9) is directly applied. Revised by the observed data, the main features of the temperature distribution in the troposphere could be well simulated although the simulation is not perfect in the lower tropical troposphere and in the upper polar troposphere. Since the EBMs do not have the capacity to directly cope with the influence of the atmospheric dynamical processes on temperature distribution, they have done their best.

4. Desertification

The United Nations Convention to Combat Desertification (UNCCD) defines the term desertification as "land degradation in arid, semi-arid, and sub-humid areas resulting from various factors including climatic variations and human activities." Seventy percent of the world's dry-lands (excluding hyper-arid deserts), or some 3600 million hectares, are degraded (UNCCD, 2000). North and Northwest China are vulnerable to climate change due to severe water shortage (Ye, 1986). By the end of 2009, total 2623700-km^2 land, accounting for 27.33% of the total land area of China, mainly the northwestern and northern areas, experienced desertification (State Forestry Administration, 2011). The drought in northern China has exacerbated in the last several decades and the arid and semiarid zones have expanded southward and eastward. Persistent drought has led to a series of local environmental problems (such as desertification), which have become a serious obstacle to economic development (Ma and Fu, 2005, 2006; Fu and Ma, 2008). How does the drought in northern China form and evolve? Answers to its eco-social impacts as well as coping strategies have become national demands. These problems have been investigated by Chinese scientists and much progress has been made, such as analysis of the three driving factors of the drought formation and development in northern China, i.e., 1) the natural law of the wet and dry variation of the monsoon system, 2) the abnormal response of the monsoon system to global change (mainly global warming), and 3) the impact of the human activity on local environment (Fu and An, 2002; Fu and Wen, 2002).

Charney (1975) analyzed the impact of vegetation at the desert margin on climate. He pointed out that a reduction of vegetation, with a consequent increase in albedo, at the southern margin of the Sahara would cause sinking motion, and additional drying, and would therefore perpetuate the arid conditions. Dickinson (1984) developed parameterization for the calculation of evapotranspiration, which distinguishes between evaporation from the ground and evapotranspiration from plant foliage. His pioneer work promoted the development and maturity of land surface models. Now, desertification is investigated mostly with the aid of land surface models or climate models, and theoretical analysis is relatively scarce. Chao and Li (2010a) hypothesized that the separating line between vegetation and desert could be determined by a given temperature value. Then, they discussed the evolution of desertification by taking albedo distribution as piecewise function (albedo values in ice, desert, and vegetation covered areas are different and discontinuous). As an application of KCCM in climate change, their work will be introduced and updated in this paper.

Although the surface albedo in different regions varies largely (Fig. 4a), its zonal averaged value shows certain rules. For example, in the Northern Hemisphere, the albedo value decreases, increases, and then decreases from north to south, corresponding to polar ice, high-latitude vegetation, midlatitude desert, and low-latitude vegetation (Fig. 4b). This characteristic is most significant in the West Asian and North American continents. In the Southern Hemisphere, the zonal mean albedo value changes similarly from south to north. The varying albedo value corresponds to Antarctic ice, sea water (the albedo value could be taken as 0.07), desert, and tropical vegetation, respectively. According to the observation data, the surface albedo distribution used in the two-dimensional model could be concluded as

$$\Gamma(x, x_s) = \begin{cases} \alpha_1(x), & 1 > x > x_s \\ \alpha_2(x), & x_s > x > x_d \\ \alpha_3(x), & x_d > x > x_v \\ \alpha_4(x), & x_v > x > 0 \end{cases} . \tag{16}$$

This means that the albedo distribution is divided into four parts according to different land surface types; $i = 1, 2, 3, 4$ means polar ice, high-latitude vegetation, midlatitude desert, and low-latitude vegetation, respectively; and x_s, x_d, x_v are the sine of the boundary latitude between polar ice, desert, and vegetation, respectively. Set $\alpha_1 = 0.75$, $\alpha_2 = \alpha_4 = 0.1$, $\alpha_3 = 0.25$, $x_s \approx 0.95$, $x_d \approx 0.766$, $x_v \approx 0.5$, and then the mean surface albedo value is around 0.15, close to the actual situation. According to the former discussion, the ice cap latitude is determined by temperature $T_s = -10$℃. Similarly, compared with the observed zonal mean temperature distribution, x_d (north edge of the desert zone) and x_v (south edge of the desert zone) are determined by $T_d = 5$℃ and $T_v = 19$℃, separately. Then, the evolution and development of desertification could be featured by the variation of the desert surface albedo value α_3 in the desert zone (see Fig. 5). In Fig. 5a, when desert surface albedo α_3 increases, both x_s (solid line) and x_d (dashed line) retreat northward, implying that polar ice sheet would shrink northward and the north edge of the desert zone would expand northward. However, the south edge of the desert zone x_v (dash-dotted line) would move northward and then southward slightly. The size variation of polar ice sheet, vegetation, and desert could change the planetary albedo of the earth (Fig. 5b). Planetary albedo Γ_p increases and then decreases with increasing desert albedo α_3. When α_3 is smaller, Γ_p would decrease with increasing α_3. This is because the increment of planetary albedo caused by α_3 itself could not balance the reduction caused by polar ice sheet shrink. The decreasing trend could last until α_3 is about 0.26. When α_3 is larger than 0.26, the reduction of albedo caused by shrinking polar ice sheet is not significant due to its smaller range. Therefore, bigger α_3 as well as its larger area would result in increasing planetary albedo. Planetary albedo change would affect the variation of temperature eventually. As shown in Fig. 5c, global integral mean surface temperature exhibits opposite changes against the planetary albedo.

Figure 6 shows the surface temperature derivation from the current climate ($\alpha_3 = 0.25$). When the

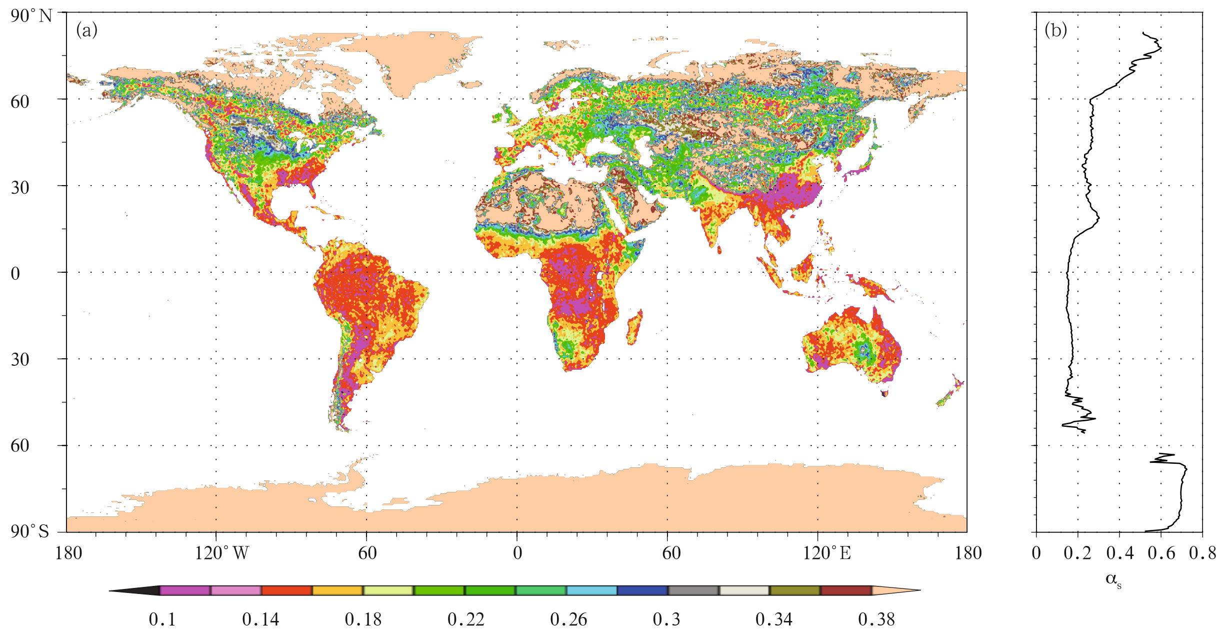

Fig. 4. (a) Spatial distribution of the surface albedo and (b) its zonal mean values. The surface albedo data are downloaded from the ESA GlobAlbedo Project website http://www.GlobAlbedo.org.

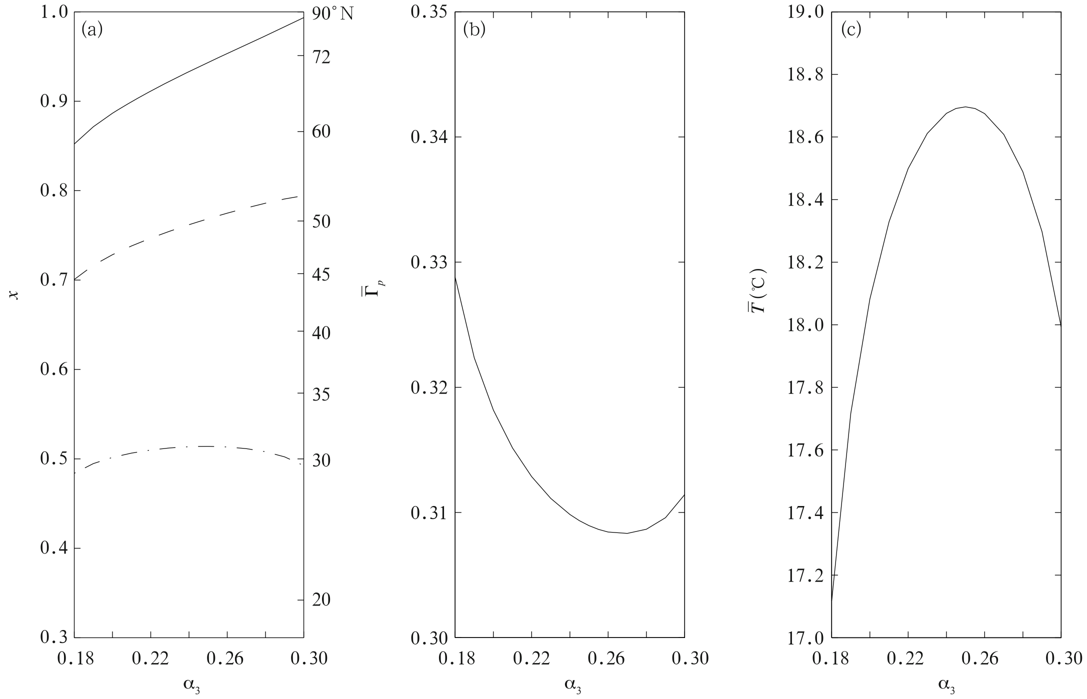

Fig. 5. Variations of (a) $x_{\rm s}$, $x_{\rm d}$, $x_{\rm v}$ (solid, dashed, and dash-dotted lines, respectively), (b) global integrated planetary albedo $\overline{\Gamma}_p$, and (c) global integrated surface temperature $\overline{T}$, with desert albedo α_3. The y-axis coordinates in (a) are latitude and its sine.

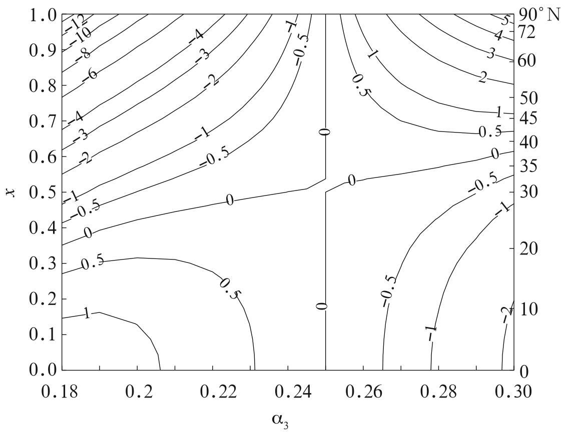

Fig. 6. Surface temperature discrepancy with the current climate states ($\alpha_3 = 0.55$). The y-axis coordinates are latitude and its sine.

desert surface albedo α_3 is smaller (larger) than present, the temperature in high latitude is lower (higher) due to larger (smaller) range of polar ice. The temperature in low latitude declines with a moderate range when α_3 is increasing. Distribution of the temperature anomalies seems to indicate that the modulation of temperature by the ice albedo feedback is more significant in high latitude.

The northward movement of the north edge of the desert zone ($x_{\rm d}$) implies that desert would invade into the high-latitude vegetation zone, which would deteriorate the local living environment, and harm the life and production of the local residents. Meanwhile the northward movement of the ice cap latitude ($x_{\rm s}$) means that high-latitude vegetation would expand closer to the polar region. Therefore, desertification would mainly shift the high-latitude vegetation zone northward rather than sharply reduce its area size. The south edge of the desert zone ($x_{\rm v}$) slightly moves northward, and then southward with increasing desert surface albedo (α_3). When the desertification is less severe, the northward movement of $x_{\rm v}$ means low-latitude vegetation would expand, which is beneficial for the local environment; when the desertification is well developed, the southward movement of $x_{\rm v}$ jeopardizes the local environment in turn. It could be seen that increase in the desert albedo would make the desert zone expand both southward and northward. Therefore, the ecological environment in the border area between desert and vegetation zone is inherently vulnerable. On the other hand, decreasing the desert albedo would reduce the area of the desert zone, hence improving the local ecological environment. The surface albedo is associated with the physical and ecological status of the underlying surface. It is necessary to study the climate sensitivity to local albedo variations. It would tell us to what extent the orderly human activities (such as irrigation and afforestation) could affect the regional and even global climate and how we could use these activities to benefit the sustainable development.

5. Warming effect of carbon dioxide

Greenhouse gases keep the mean surface temperature of our earth in a suitable range by absorbing the upward longwave radiation. Excessive greenhouse gases would warm the earth. The carbon dioxide emitted by human production and industrial activities is thought to be an important element driving the global warming. The concentration of carbon dioxide in the atmosphere has been steadily increasing since the Industrial Revolution. According to a report by the World Meteorology Organization (WMO), the daily average concentration of carbon dioxide has broken 400 ppm at several stations, climbing up the historical maximum record in the past 3 million years (WMO, 2013). At the current increasing rate, the annual mean concentration of carbon dioxide would exceed 400 ppm in 2014 or 2015. It is widely believed that with the increasing concentration of carbon dioxide, the global average surface temperature will continue to rise. However, it seems inconclusive about the rate of the surface temperature increase, especially during the last decade when steady increasing concentration of carbon dioxide did not correspond to a steady warming globe; instead, the global mean temperature shows a so-called "hiatus" (Kerr, 2009; Knight et al., 2009). Therefore, the relationship between concentration of carbon dioxide and global temperature variations is still obscure and various research methods and angles are still necessary.

Liu et al. (2002) discussed the saturation of greenhouse effect due to atmospheric carbon dioxide. They pointed out that absorption has saturated in the center of the carbon dioxide 15-μm band indeed. However, absorption does not saturate for the wings of the 15-μm band and the other bands in the near future. This indicates that potential warming effect of carbon dioxide still cannot be ignored. Climate models are useful tools to investigate the relationship between concentration of carbon dioxide and temperature variation. Meanwhile, some simple models with solid physical processes are useful supplements that could be used to deepen our understanding about the issue. For example, Liu (2006) analyzed the physical processes controlling the glacial and interglacial cycles during the past several hundred thousand years by using the simple, one-dimensional Budyko model (Budyko, 1969).

Chao and Li (2010b) discussed the response of atmospheric temperature to concentration of carbon dioxide with the KCCM. Here, more reasonable physical consideration is added to update their work. The atmospheric absorption coefficient k_j for longwave radiation could be written as

$$k_j = \eta(\alpha_{H_2O,j}\rho_{H_2O} + \alpha_{CO_2,j}\rho_{CO_2}), \quad (17)$$

where $\eta = 5/3$ is the scatter factor, α is the mass absorption coefficient, ρ is the concentration of absorbing medium, subscripts H_2O and CO_2 represent vapor and carbon dioxide. The absorption coefficient in longwave band (4–100 μm) could be obtained from HITRAN ON THE WEB website (http://hitran.iao.ru/) which calculates the absorption coefficient according to HITRAN 2004 high-resolution spectral data. Absorption coefficients in different bands can vary several orders of magnitude, which is very complex for theoretical analysis. For convenience, the absorption simplification scheme in longwave bands used by Kuo (1973) is still adopted, in which the entire long-wave band is divided into strong and weak absorption zones.

The variations of absorption coefficients in strong and weak absorption zones with concentration of carbon dioxide show different features (Fig. 7). The absorption coefficient in the strong absorption zone will decline gradually with increasing concentration of carbon dioxide until 400 ppmv, higher than which it decreases slightly and then even increases slightly when concentration is higher than 600 ppmv (dash-dotted line). The absorption coefficient in the weak absorption zone varies dramatically when concentration of carbon dioxide is lower than 200 ppmv, higher than which it decreases overall but with many small fluctuations (solid line). Although both strong and weak absorption coefficients decrease overall, the energy proportion absorbed by the strong absorption zone increases steadily with increasing concentration of carbon dioxide (dashed line). Chao and Li (2010b) hypothesized that the strong and weak absorption zone each absorbs half of the entire energy. Therefore, the proportion equals 0.5. This assumption is a little coarse albeit simple and intuitive. As an improvement to the previous work, more accurate HITRAN 2004 high-resolution spectral data are used here to calculate the strong and weak absorption coefficients according to Eq. (8). The coefficients vary with air temperature and pressure but with no obvious discrepancy in the calculation of air temperature distribution, which could be close to the observed data through adjusting the turbulent exchange coefficients.

Referring to the former discussion, the surface albedo is divided into two parts, namely, $\alpha_1 = 0.62$ and $\alpha_2 = 0.132$. The variation of ice cap latitude with concentration of carbon dioxide is then calculated (Fig. 8). In the equilibrium representing the current climate (Fig. 8a), ice cap retreats northward slowly with many small fluctuations when concentration of carbon dioxide is increasing. Ice cap latitude varies little in some concentration ranges such as 400–500 ppmv, and even moves southward slightly in some ranges such as 900–950 ppmv. Polar ice would melt completely (ice-free earth) only when the concentration increases to about 1000 ppmv. Global mean surface temperature increases gradually but with a slowing rate in the current climate (Fig. 9a). When concentration is higher than 600 ppmv, there is almost no temperature rise, that is, the "saturation" state. In the equilibrium representing the glacial climate, ice cap latitude would march southward gradually with increasing concentration of carbon dioxide (Fig. 8b). Temperature decreases in the glacial climate (Fig. 9b). The mean surface temperature could be even lower than −10°C when carbon dioxide exceeds 600 ppmv. Carbon dioxide does not act as a driving factor for global warming. Oppositely, it accompanies with global cooling. In the equilibrium representing "snowball earth", the ice cap latitude is situated in the tropics. Polar ice would shrink its range when higher concentration of carbon dioxide is added into the atmosphere (Fig. 8c). Even though temperature is as low as about −55°C in the snowball earth, it would still decline with increasing content of carbon dioxide (Fig. 9c). The discrepancy in the three equilibrium climatic states might be

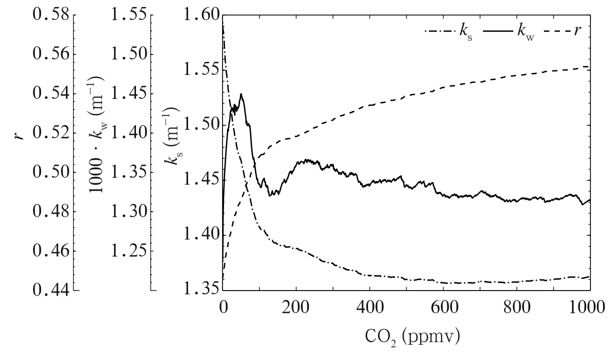

Fig. 7. Absorption coefficients in strong and weak absorptive zones and the proportion of the strong absorptive zone in the total absorption as a function of the concentration of carbon dioxide. The reference temperature and pressure are set to 290 K and the standard atmospheric pressure.

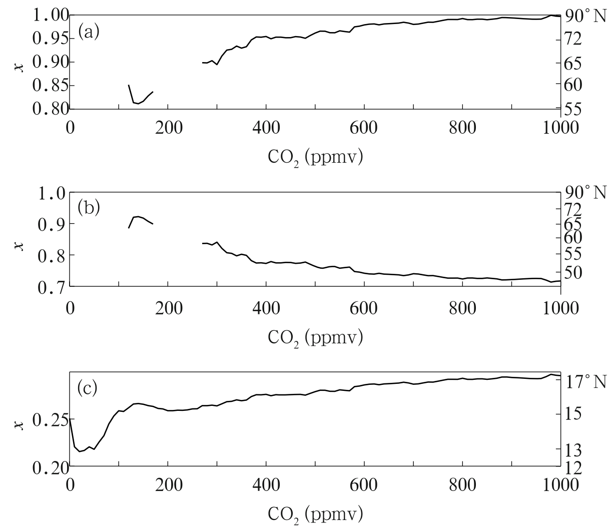

Fig. 8. Variations of the three equilibrium ice cap latitudes with the change of carbon dioxide concentration.

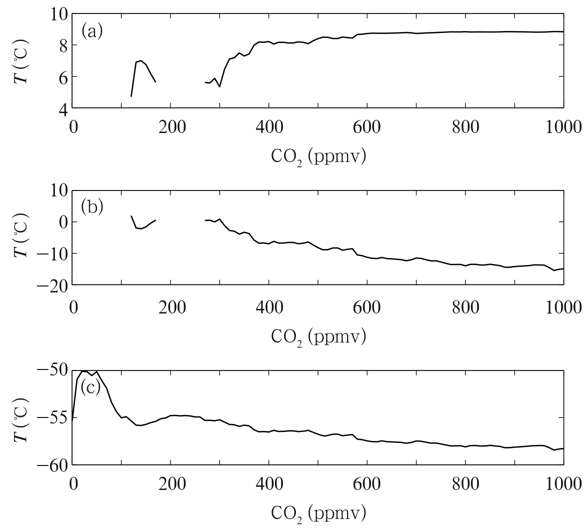

Fig. 9. Variations of the global mean surface temperature in the three equilibrium states with the change in concentration of carbon dioxide.

caused by the relative importance of albedo feedback and greenhouse effect. The climate would tend to be cooling if albedo feedback overweighs the greenhouse effect, and be warming if greenhouse effect dominates. It should be noted that the missing values of ice cap latitude in the current and glacial climates suggest that no stable ice cap latitude correspond to the CO_2 concentration ranges of 0–120 ppmv and 180–270 ppmv, in which the two equilibrium states converge together and there is no real roots (ice cap latitudes) for the system.

6. Concluding remarks

The development of two-dimensional EBM and its application to some climatic issues have been systematically reviewed in this paper. Though simple, two-dimensional EBM has a solid physical basis. It could deepen the understanding of climate change and supplement the results from numerical climate models. The climatic issues discussed in this paper are of great concern currently, but not yet completely understood. The answers to these issues are likely to facilitate certain progress and breakthroughs in the future, which to some extent represents the direction of theoretical research in the climate change field. Besides, paleoclimatic data and climate models are needed to further verify the theoretical results. We hereby propose that future topics in these regards include but are not limited to the following.

(1) The relationship between albedo distribution and ice cap latitude equilibria as well as the stability of the equilibria. The numbers of equilibrium are determined by the continuity of the surface albedo distribution. Smoother albedo helps reduce the equilibrium numbers, and vice versa. The actual albedo distribution is continuous in some regions but discontinuous in other regions. It is still unclear which situation is more likely to occur. In addition, questions such as how the equilibrium stability is, whether unstable equilibrium has any physical meaning, and how the unstable equilibrium evolves, all need further study. The answers to these questions would help to understand which state the climate system is currently evolving in and its development direction.

(2) Desertification and its impact on the global temperature distribution and atmospheric motion. Current researches mainly focus on the impact of surface albedo distribution on desertification and air motion in the planetary boundary layer. It is necessary to analyze the impact of desertification on atmospheric circulation by coupling the two-dimensional EBM with the atmospheric motion equations in the free atmosphere. Meanwhile, it is still simple and certainly artificial to fix the albedo distribution and the south/north edge of the desert zone; it is still unclear how sensitive the desertification is to these parameters; it is still unknown how to appropriately define the land surface characteristics that can both reflect the physical nature and facilitate the theoretical calculation. All these questions need to be addressed.

(3) The impact of concentration of carbon dioxide on global mean surface temperature. According to the theoretical calculation, the relationship between concentration of carbon dioxide and global mean surface temperature is not monotonic. Increasing carbon dioxide might correspond to upward or downward temperature trend. Temperature increment caused by CO_2 concentration increase tends to become zero (or called warming saturation) in the current climate state. It is still uncertain about whether such a conclusion is rea-

sonable or not, and whether such a phenomenon could occur or not in more realistic land cover distribution. All these equations deserve further study.

Besides the three questions discussed above, two-dimensional EBM could also be used on other important climatic issues, such as the climatic effect of aerosols and so on.

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