SCIENCE CHINA
Earth Sciences

• RESEARCH PAPER •

March 2011 Vol.54 No.3: 420–430
doi: 10.1007/s11430-010-4050-8

Prediction of water table depths under soil water-groundwater interaction and stream water conveyance

DI ZhenHua[1,2], XIE ZhengHui[1*], YUAN Xing[1], TIAN XiangJun[1], LUO ZhenDong[3] & CHEN YaNing[4]

[1] *ICCES/LASG, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China;*
[2] *School of Science, Beijing Jiaotong University, Beijing 100044, China;*
[3] *Department of Mathematics & Physics, North China Electric Power University, Beijing 102206, China;*
[4] *Key Laboratory of Oasis Ecology and Desert Environment, Xinjiang Institute of Ecology and Geography, Chinese Academy of Sciences, Urumqi 830011, China*

Received April 30, 2009; accepted January 17, 2010; published online October 12, 2010

Water table over an arid region can be elevated to a critical level to sustain terrestrial ecosystem along the natural channel by the stream water conveyance. Estimation of water table depth and soil moisture on river channel profile may be reduced to a two-dimensional moving boundary problem with soil water-groundwater interaction. The two-dimensional soil water flow with stream water transferred is divided into an unsaturated vertical soil water flow and a horizontal groundwater flow. Therefore, a prediction model scheme for water table depths under the interaction between soil water and groundwater with stream water transferred is presented, which includes a vertical soil water movement model, a horizontal groundwater movement model, and an interface model. The synthetic experiments are conducted to test the sensitivities of the river elevation, horizontal conductivity, and surface flux, and the results from the experiments show the robustness of the proposed scheme under different conditions. The groundwater horizontal conductivity of the proposed scheme is also calibrated by SCE-UA method and validated by data collected at the Yingsu section in the lower reaches of the Tarim River, which shows that the model can reasonably simulate the water table depths.

stream water conveyance, soil water-groundwater interaction model, SCE-UA method

Citation: Di Z H, Xie Z H, Yuan X, et al. Prediction of water table depths under soil water-groundwater interaction and stream water conveyance. Sci China Earth Sci, 2011, 54: 420–430, doi: 10.1007/s11430-010-4050-8

Groundwater over arid regions is one of the most important ecological factors. The change of water table over a region influences soil moisture at the surface directly and then the growth and development of natural vegetation over the region. For regions with deep water table, groundwater on the river sides along a river can be recharged laterally from stream water, and water table can be elevated to a critical depth to make vegetation get enough soil moisture, and thus to sustain the ecological balance of riparian zone [1–4]. The key issue of elevating water table reasonably is how to accurately predict the water table depth under the interaction between soil water and groundwater with stream water transferred and then to estimate from the prediction the basic water requirement for the channel and the effective duration of water conveyance, which is important for water resources management.

Several studies discussed soil water flow problem. For example, Xie et al. [5] developed an unsaturated soil water flow numerical model based on the mass-lumped finite element method, and Luo et al. [6] and Xie et al. [7] presented another numerical model to compute soil moisture and wa-

*Corresponding author (email: zxie@lasg.iap.ac.cn)

ter flow flux together by means of a mixed finite element method, which neglected dynamic variation of water table. Water table dynamics have been also accounted for in several studies. Yuan et al. [8] developed an estimation method of water table depths in large scale and applied it to the whole continental China based on relationship between infiltration rates and water table depths. Chen et al. [9] established a statistic model to estimate water table based on relationship between river discharges and water table depths. Xie et al. [10] proposed a statistical-dynamical scheme derived from the Darcy's law with the relationship between river elevations and water table depths to estimate water table depths. However, these studies did not consider the interaction between soil water and groundwater. On the basis of the work in Xie et al. [5], Liang et al. [11, 12] presented a parameterization to represent soil water and groundwater interaction dynamics for land surface model through reducing the interaction problem into a moving boundary problem, and Yang et al. [13] reduced the moving boundary problem to a fixed boundary problem by coordinate transformation to solve problem numerically. However, these works neglected the influence of groundwater lateral flow.

In this work, the problem of soil water and groundwater interaction with stream water conveyance at river cross section is reduced to a two-dimensional moving boundary problem, and a new quasi two-dimensional framework for water table prediction is developed through dividing the two-dimensional soil water flow into an unsaturated vertical soil water flow and a horizontal groundwater flow. Sensitivity experiments are conducted to analyze main parameters of model. In the end, the proposed framework is validated by observation data collected at the Yingsu section in the lower reaches of the Tarim River.

1 Soil water-groundwater interaction model with stream water transferred

In this section, we first introduce a two-dimensional theory model of the interaction between soil water and groundwater with stream water transferred, and then reduce it to a quasi two-dimensional model. Finally, numerical algorithms for the quasi two-dimensional model are presented.

1.1 The two-dimensional moving boundary problem of the interaction between soil water and groundwater with stream water transferred

With stream water transferred, river water leaking from river bed move around the channel for water potential differences, which makes the soil zone around the channel saturated first. So, three zones appear from river bed to the impervious bedrock after a period of time. They are the saturated soil water zone Ω_1, the unsaturated soil water zone Ω_2, and groundwater zone Ω_3. Without regard to soil water movement in parallel with natural channel, we reduce soil water and groundwater interaction to a two-dimensional moving boundary problem on the vertical profile of river channel.

Figure 1 shows soil profile for leakage between stream and aquifer at the beginning of stream water conveyance. In the saturated soil water zone Ω_1, the water balance equation for water potential $\psi[L]$ can be written as

$$S_s \frac{\partial \psi}{\partial t} - \nabla \cdot \left(K_s \nabla \psi \right) - g = 0 , \tag{1}$$

where $S_s[1/L]$ is the specific storativity, $K_s[L/T]$ is the saturated hydraulic conductivity, and $g[1/T]$ is a sink term.

In the unsaturated soil water zone Ω_2, the water balance equation for soil moisture $\theta[L^3/L^3]$ can be written as

$$\frac{\partial \theta}{\partial t} - \nabla \cdot \left(D(\theta) \nabla \theta \right) - \frac{\partial K(\theta)}{\partial z} = g , \tag{2}$$

where $D(\theta)[L^2/T]$ is the hydraulic diffusivity, $K(\theta)[L/T]$ is the unsaturated hydraulic conductivity, and $g[1/T]$ is the sink term.

On the two-dimensional moving boundary Γ_1, let $V_n[L/T]$ represent the flow rate in normal direction, we have

$$V_n = (q_s - q_u) \cdot n(t) , \tag{3}$$

where $q_s = -K_s \nabla \psi$, $q_u = -D(\theta)\nabla\theta$, $q_s[L/T]$ is flux in zone Ω_1, $q_u[L/T]$ is flux in zone Ω_2, and $n(t)$ is the unit external normal vector on the boundary Γ_1. The eqs. (1)–(3) with initial and boundary conditions are formulated into the two-dimensional moving boundary problem.

1.2 A quasi two-dimensional model framework of the interaction between soil water and groundwater with stream water transferred

In practice, the moving boundary is changing with the river elevation. When the lateral flow of groundwater with water conveyance is considered, river water can arrive at the water table in a short period of time. Therefore, in the case of shallow water table or long time persistence of stream, soil

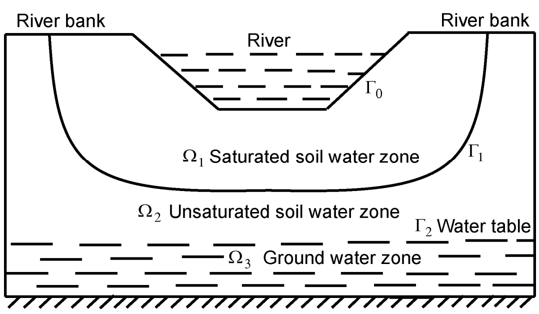

Figure 1 Soil profile for leakage between stream and aquifer at the beginning of stream water conveyance.

moisture around the bedrock is saturated, and we assume that river water is water table (see Figure 2). Dynamic variation of water table can be impacted through the vertical flux on the moving boundary and the horizontal flux from groundwater lateral flow. In fact, the model validation can show reasonability of the assumption.

As shown in Figure 2, x axis denotes the impervious bedrock, z axis denotes the river bank that is perpendicular to the impervious bedrock, H is the height from the impervious bedrock to the surface, and L is the level length for the study. The saturated groundwater zone lies on the impervious bedrock, and a moving boundary Γ_1 separates the soil profile into the unsaturated zone above and the saturated zone below.

Due to the effect of gravity in the unsaturated zone, there is little lateral flow, and soil water flow mainly occurs in vertical direction. Groundwater flow in the saturated zone is approximated as horizontal flow. The horizontal flow column [0, L] is divided into n grid cells such that $I_1, I_2, \cdots, I_n$, and then there are n vertical soil columns in the unsaturated zone. As described in Figure 2 for each soil column $x \in I_i$ (i=1, 2, ⋯, n), the unsaturated vertical soil water flow equation, with initial and boundary conditions [14–16], can be rewritten as follows:

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right]+\frac{\partial K(\theta)}{\partial z}+f(x,z,t),$$

$$h(x,t)<z<H, \tag{4}$$

$$\theta(x,z,0)=\theta_0(x,z),\ \ h_0(x)<z\leqslant H, \tag{5}$$

$$q_{z=H}(x,t)=P-E-R,\ \ z=H,\ \ t>0, \tag{6}$$

$$\theta=\theta_s,\ z=h(x,t),\ \ t>0, \tag{7}$$

where $f(x, z, t)[1/T]$ is a sink term, $h(x, t)[L]$ is water table elevation for the ith grid cell at time $t[T]$, $q_{z=H}(x,t)[L/T]$ is the flux across the surface (i.e., $z = H$ at the surface), $P[L/T]$ is the rainfall reaching the soil surface, $E[L/T]$ is ground evaporation, $R[L/T]$ is surface runoff, $\theta_s[L^3/L^3]$ is the saturated soil moisture, $h_0[L]$ and $\theta_0(x, z)[L^3/L^3]$ are initial values of the water table elevation and soil moisture, respectively. $D(\theta)[L^2/T]$ is the hydraulic diffusivity, $K(\theta)[L/T]$ is the unsaturated hydraulic conductivity, and z represents the vertical direction and is assumed positive upward.

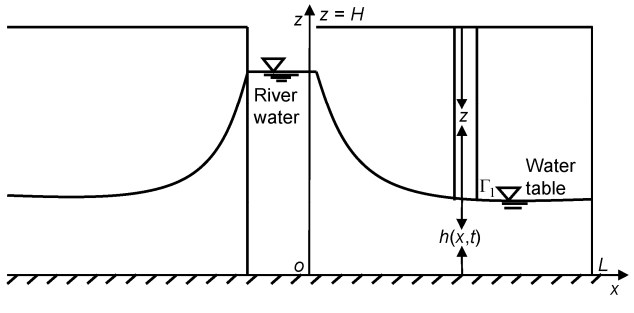

Figure 2 A schematic representation of lateral flow.

For the horizontal groundwater flow column [0, L](i.e., saturated zone), integrating saturated water eq. (1) from the impervious bedrock to the water table and averaging it, based on Dupuit approximation [17], we have

$$n_e\frac{\partial h}{\partial t}=\frac{\partial}{\partial x}\left(K_s h\frac{\partial h}{\partial x}\right)-q_{z=h(x,t)}(x,t),$$

$$0<x<L,\, 0<z\leqslant h(x,t). \tag{8}$$

The corresponding initial and boundary conditions are expressed as follows:

$$h(x,0)=h_0(x),\ \ 0\leqslant x\leqslant L,\ \ t=0, \tag{9}$$

$$h(0,t)=h_r,\ \ t>0, \tag{10}$$

$$q_{x=L}(L,t)=0,\ x=L,\ \ t>0, \tag{11}$$

where $h(x, t)[L]$ is water table elevation, and is also the mean water potential at the cross section of horizontal groundwater flow, for $x[L]$ from the river bank at time $t[T]$. $K_s[L/T]$ is the saturated hydraulic conductivity, $n_e[L^3/L^3]$ is the specific yield, $q_{z=h(x,t)}(x, t)[L/T]$ assumed positive upward is exchange flux between unsaturated zone and saturated zone on the moving boundary, and $h_r[L]$ and $h_0(x)[L]$ are the river elevation and initial water table elevation, respectively.

Integrating eq. (4) over unsaturated soil elevation ($h(x, t)$, H) and using $q_z(x,t)=-D(\theta)\frac{\partial \theta}{\partial z}-K(\theta)$, we have

$$q_{z=h(x,t)}(x,t)=\int_{h(x,t)}^{H}\frac{\partial \theta}{\partial t}\mathrm{d}z+q_{z=H}(x,t),\ x\in I_i. \tag{12}$$

Eq. (12) is connecting soil water flow problem (eqs. (4)–(7)) with groundwater flow problem (eqs. (8)–(11)).

1.3 Numerical algorithms for the quasi two-dimensional model

The quasi two-dimensional model presented in section 1.2 is solved numerically by applying finite difference methods in time and space. First, the algorithms for one-dimensional unsaturated vertical soil water model, one-dimensional saturated horizontal groundwater model, and their connection equation are presented, respectively. Second, a numerical procedure for the quasi two-dimensional model is presented. Then the estimation scheme of water table depths, based on the quasi two-dimensional model and a parameter calibration method SCE-UA, is also included in this section.

1.3.1 Discretization of the one-dimensional unsaturated vertical soil water model

For each unsaturated soil column $x\in I_i$ (i=1, 2,⋯, n), the column is divided into m layers from the ground surface to the water table, and each layer thickness is $\Delta z_1, \Delta z_2, \cdots, \Delta z_m$, respectively. Three soil layers j–1, j, and j+1 are shown in Figure 3. Soil moisture is defined at the layer node depth z_j (j=1, 2, ⋯, m), which is middle value of the interface depth $z_{j-\frac{1}{2}}$ and $z_{j+\frac{1}{2}}$. The hydraulic diffusivity $D(\theta)$, the unsaturated hydraulic conductivity $K(\theta)$, and soil water flux q are defined at the interface depth, respectively. The soil water flux q is assumed positive upward.

For time step Δt, the unsaturated soil water discretion equations with m layers can be written as follows:

$$\frac{\theta_j^{k+1}-\theta_j^k}{\Delta t}=\frac{D_{j+\frac{1}{2}}^{k+1}\left(\frac{\theta_{j+1}^{k+1}-\theta_j^{k+1}}{z_{j+1}-z_j}\right)+K_{j+\frac{1}{2}}^{k+1}+q_{j-\frac{1}{2}}^{k+1}}{\Delta z_j},\ j=1, \qquad (13)$$

$$\frac{\theta_j^{k+1}-\theta_j^k}{\Delta t}=\frac{D_{j+\frac{1}{2}}^{k+1}\left(\frac{\theta_{j+1}^{k+1}-\theta_j^{k+1}}{z_{j+1}-z_j}\right)-D_{j-\frac{1}{2}}^{k+1}\left(\frac{\theta_j^{k+1}-\theta_{j-1}^{k+1}}{z_j-z_{j-1}}\right)}{\Delta z_j}+\frac{K_{j+\frac{1}{2}}^{k+1}-K_{j-\frac{1}{2}}^{k+1}}{\Delta z_j},\quad j=2, 3, \cdots, m-1. \qquad (14)$$

By adding a layer that has the same thickness with the mth layer (i.e., called the m+1th layer) in saturated zone, the discretion equation of the mth layer, based on eq. (4) and the saturated groundwater boundary condition, can be obtained as

$$\frac{\theta_j^{k+1}-\theta_j^k}{\Delta t}=\frac{D_{j+\frac{1}{2}}^{k+1}\left(\frac{\theta_s-\theta_j^{k+1}}{z_{j+1}-z_j}\right)-D_{j-\frac{1}{2}}^{k+1}\left(\frac{\theta_j^{k+1}-\theta_{j-1}^{k+1}}{z_j-z_{j-1}}\right)}{\Delta z_j}+\frac{K_{j+\frac{1}{2}}^{k+1}-K_{j-\frac{1}{2}}^{k+1}}{\Delta z_j},\quad j=m. \qquad (15)$$

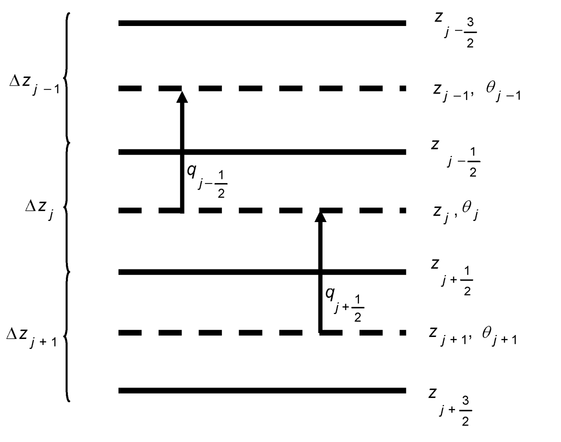

Figure 3 A schematic representation of numerical scheme used to solve for soil moisture. Three soil layers j–1, j, and j+1 are shown, z_{j-1}, z_j, and z_{j+1} are the layer node depths, and $z_{j-\frac{3}{2}}$, $z_{j-\frac{1}{2}}$, $z_{j+\frac{1}{2}}$, and $z_{j+\frac{3}{2}}$ are the interface depths.

Let

$$a_1=0,\ b_1=1+\frac{\Delta t D_{\frac{3}{2}}^{k+1}}{\Delta z_1\left(z_2-z_1\right)},\ c_1=-\frac{\Delta t D_{\frac{3}{2}}^{k+1}}{\Delta z_1\left(z_2-z_1\right)},$$

$$h_1=\theta_1^k+\frac{\Delta t}{\Delta z_1}\left(K_{\frac{3}{2}}^{k+1}+q_{\frac{1}{2}}^{k+1}\right);\quad a_j=-\frac{\Delta t D_{j-\frac{1}{2}}^{k+1}}{\Delta z_j(z_j-z_{j-1})},$$

$$b_j=1+\frac{\Delta t D_{j+\frac{1}{2}}^{k+1}}{\Delta z_j(z_{j+1}-z_j)}+\frac{\Delta t D_{j-\frac{1}{2}}^{k+1}}{\Delta z_j(z_j-z_{j-1})},\quad c_j=-\frac{\Delta t D_{j+\frac{1}{2}}^{k+1}}{\Delta z_j(z_{j+1}-z_j)},$$

$$h_j=\theta_j^k+\frac{\Delta t}{\Delta z_j}\left(K_{j+\frac{1}{2}}^{k+1}-K_{j-\frac{1}{2}}^{k+1}\right), j=2, 3, \cdots, m-1;$$

$$a_m=-\frac{\Delta t D_{m-\frac{1}{2}}^{k+1}}{\Delta z_m(z_m-z_{m-1})},$$

$$b_m=1+\frac{\Delta t D_{m+\frac{1}{2}}^{k+1}}{\Delta z_m(z_{m+1}-z_m)}+\frac{\Delta t D_{m-\frac{1}{2}}^{k+1}}{\Delta z_m(z_m-z_{m-1})},$$

$$h_m=\theta_m^k+\frac{\Delta t D_{m+\frac{1}{2}}^{k+1}\theta_s}{\Delta z_m(z_{m+1}-z_m)}+\frac{\Delta t}{\Delta z_m}\left(K_{m+\frac{1}{2}}^{k+1}-K_{m-\frac{1}{2}}^{k+1}\right).$$

Thus, the eqs. (13)–(15) are expressed as follows:

$$\begin{cases} b_1\theta_1^{k+1}+c_1\theta_2^{k+1}=h_1, \\ a_j\theta_{j-1}^{k+1}+b_j\theta_j^{k+1}+c_j\theta_{j+1}^{k+1}=h_j,\quad j=2, 3, \cdots, m-1, \\ a_m\theta_{m-1}^{k+1}+b_m\theta_m^{k+1}=h_m. \end{cases} \qquad (16)$$

Discretization of the eq. (4) in a unsaturated soil column $x\in I_i$ (i=1, 2, ⋯, n) is presented above, and the discrete equations for other soil columns are formulated in a similar way.

1.3.2 Discretization of the one-dimensional saturated horizontal groundwater model

Based on the discussion in section 1.2, the horizontal domain [0, L] is divided into n grid cells, such that I_1, I_2, ⋯, I_n, and the cell lengths related to the grid cells are Δx_1, Δx_2, ⋯, Δx_n, respectively. Each cell node is the mediate position of cell, and the distances between the cell nodes and river bank are x_1, x_2, ⋯, x_n, respectively. Water table elevations on the cell nodes and cell interfaces are defined as h_i (i=1, 2, ⋯, n) and $h_{i+\frac{1}{2}}$ (i=1, 2, ⋯, n), respectively. The time step is Δt.

For each I_i (i=1, 2, ⋯, n), differential eq. (8) with boundary conditions can be written as follows.

The difference equation for the first grid cell, with the first boundary condition $h(0, t) = h_r$, can be written as follow:

$$\frac{n_e h_i^{k+1}-n_e h_i^k}{\Delta t}=\frac{K_s h_{i+\frac{1}{2}}^{k+1}\left(\frac{h_{i+1}^{k+1}-h_i^{k+1}}{x_{i+1}-x_i}\right)-K_s h_{i-\frac{1}{2}}^{k+1}\left(\frac{h_i^{k+1}-h_r}{x_i}\right)}{\Delta x_i}$$

$$-q^{k+1}_{z=h_i^k}(x_i,t), \quad i=1. \tag{17}$$

The difference equations for the internal cell I_i (i=2, ⋯, n−1) can be written as follow:

$$\frac{n_e h_i^{k+1}-n_e h_i^k}{\Delta t}=\frac{K_s h^{k+1}_{i+\frac{1}{2}}\left(\frac{h_{i+1}^{k+1}-h_i^{k+1}}{x_{i+1}-x_i}\right)-K_s h^{k+1}_{i-\frac{1}{2}}\left(\frac{h_i^{k+1}-h_{i-1}^{k+1}}{x_i-x_{i-1}}\right)}{\Delta x_i}$$

$$-q^{k+1}_{z=h_i^k}(x_i,t), \quad i=2,3,\cdots,n-1. \tag{18}$$

The difference equation for the nth cell, with the second boundary condition q=0, can be obtained as follows through adding a cell with the same length with the nth cell (i.e., called the n+1th cell):

$$\begin{cases}\dfrac{h_{n+1}^{k+1}-h_n^{k+1}}{x_{n+1}-x_n}=0,\\ \dfrac{n_e h_n^{k+1}-n_e h_n^k}{\Delta t}\\ =\dfrac{K_s h^{k+1}_{n+\frac{1}{2}}\left(\frac{h_{n+1}^{k+1}-h_n^{k+1}}{x_{n+1}-x_n}\right)-K_s h^{k+1}_{n-\frac{1}{2}}\left(\frac{h_n^{k+1}-h_{n-1}^{k+1}}{x_n-x_{n-1}}\right)}{\Delta x_n}\\ -q^{k+1}_{z=h_n^k}(x_n,t).\end{cases} \tag{19}$$

Let $h^{k+1}_{n+\frac{1}{2}}\approx h_n^{k+1}$ and $\alpha_1=-\dfrac{K_s\Delta t h^{k+1}_{\frac{1}{2}}}{x_1\Delta x_1}$,

$$\beta_1=n_e+\frac{\Delta t K_s h^{k+1}_{\frac{3}{2}}}{\Delta x_1(x_2-x_1)}+\frac{\Delta t K_s h^{k+1}_{\frac{1}{2}}}{x_1\Delta x_1},$$

$$\gamma_1=-\frac{\Delta t K_s h^{k+1}_{\frac{3}{2}}}{\Delta x_1(x_2-x_1)}, \quad \delta_1=n_e h_1^k-q^{k+1}_{z=h_1^k}(x_1,t)\Delta t-\alpha_1 h_r;$$

$$\alpha_i=-\frac{K_s\Delta t h^{k+1}_{i-\frac{1}{2}}}{\Delta x_i(x_i-x_{i-1})},$$

$$\beta_i=n_e+\frac{\Delta t K_s h^{k+1}_{i+\frac{1}{2}}}{\Delta x_i(x_{i+1}-x_i)}+\frac{\Delta t K_s h^{k+1}_{i-\frac{1}{2}}}{\Delta x_i(x_i-x_{i-1})},$$

$$\gamma_i=-\frac{\Delta t K_s h^{k+1}_{i+\frac{1}{2}}}{\Delta x_i(x_{i+1}-x_i)}, \quad \delta_i=n_e h_i^k-q^{k+1}_{z=h_i^k}(x_i,t)\Delta t,$$

$i=2,3,\cdots,n-1;$

$$\alpha_n=-\frac{\Delta t K_s h^{k+1}_{n-\frac{1}{2}}}{\Delta x_n(x_n-x_{n-1})}, \quad \beta_n=n_e+\frac{\Delta t K_s h^{k+1}_{n-\frac{1}{2}}}{\Delta x_n(x_n-x_{n-1})},$$

$$\delta_n=n_e h_n^k-q^{k+1}_{z=h_n^k}(x_n,t)\Delta t.$$

Finally, discretion equations for groundwater flow model can be expressed as follows:

$$\begin{cases}\beta_1 h_1^{k+1}+\gamma_1 h_2^{k+1}=\delta_1,\\ \alpha_i h_{i-1}^{k+1}+\beta_i h_i^{k+1}+\gamma_i h_{i+1}^{k+1}=\delta_i, \quad i=2,3,\cdots,n-1,\\ \alpha_n h_{n-1}^{k+1}+\beta_n h_n^{k+1}=\delta_n.\end{cases} \tag{20}$$

1.3.3 Discretization of the connection equation

In each horizontal grid cell $x\in I_i$ (i=1, 2, ⋯, n), discretization of connection eq. (12) (i.e., moving boundary) can be expressed as

$$q^{k+1}_{z=h_i^k}(x_i,t)=\frac{1}{\Delta t}\sum_{j=1}^{m}\left[\theta_j^{k+1}(x_i)-\theta_j^k(x_i)\right]\Delta z_j+q^{k+1}_{z=H}(x_i,t). \tag{21}$$

1.3.4 Numerical procedure for the quasi two-dimensional model

On the basis of the developed quasi two-dimensional numerical model, we now describe how to dynamically compute soil moisture and water table elevations at next time $t+\Delta t$ with respective known values at time t.

The numerical procedure can be briefly summarized as follows:

1) Assume that the soil moisture $\theta(x, z, t)$ and the water table elevation $h(x, t)$ are given at time t. Fix $h(x_i, t)$ in horizontal grid cell I_i (i=1, 2, ⋯, n), and then compute pre-estimate value $\theta'(x_i, z, t+\Delta t)$ with eq. (16) from $\theta(x_i, z, t)$ and $h(x_i, t)$. Iterate on $\theta'(x_i, z, t+\Delta t)$ until it converges.

2) Compute pre-estimate value $q'_{z=h(x_i,t)}(x_i, t+\Delta t)$ of flux across the moving boundary in the horizontal grid cell I_i with eq. (21) from $\theta(x_i, z, t)$, $\theta'(x_i, z, t+\Delta t)$ in step 1, and provided surface flux.

3) Repeat steps 1)–2) in all horizontal grid cells I_i (i=1, 2, ⋯, n), and then put all $q'_{z=h(x_i,t)}(x_i, t+\Delta t)$ (i=1, 2, ⋯, n) into eq. (20) to compute pre-estimate value $h'(x_i, t+\Delta t)$(i=1, 2, ⋯, n) on the basis of $h(x_i, t)$ (i=1, 2, ⋯, n). Iterate on $h'(x_i, t+\Delta t)$(i=1, 2, ⋯, n) until they converge.

4) With updated $h'(x_i, t+\Delta t)$ and$\theta'(x_i, z, t+\Delta t)$ (i=1, 2, ⋯, n) from the steps 1)–3) as iterative initial values, repeat steps 1)–3) until $h'(x_i, t+\Delta t)$ (i=1, 2, ⋯, n) (i.e., $h'(x, t+\Delta t)$) converge. If the convergence happens, $h'(x, t+\Delta t)$ and $\theta'(x, z, t+\Delta t)$ are correct values of water table elevations and soil moisture at time $t+\Delta t$, respectively;

5) Repeat the steps 1)–4) for the next time step.

1.3.5 An estimation scheme of water table depth based on model GSIM and parameter calibration method SCE-UA

On the basis of the developed soil water-groundwater interaction model GSIM, we estimate groundwater horizontal hydraulic conductivity K_s, which plays an important role in the prediction for water table depths, by applying the Shuffled Complex Evolution method developed at the University of Arizona (SCE-UA). The SCE-UA algorithm is a global optimization method proposed by Duan et al. [18–20] for optimizing automatically parameters of conceptual rainfall runoff models with an interval constraint, nonlinear, multiextremum characteristics, and without a specific function expression, and is now widely used to hydrological forecasting. The water table elevation estimation scheme, based on SCE-UA, is shown in Figure 4. Water table depth can be easily obtained from the difference between the surface elevation and water table elevation.

2 Model validations

2.1 Sensitivities of model parameters

In order to test the developed soil water-groundwater interaction model, the sensitivity analyses of model parameters are conducted first. There are three important model parameters: river elevation h_r, groundwater horizontal hydraulic conductivity K_s, and surface flux $P–E–R$, where P is the rainfall reaching the soil surface, E is ground evaporation, and R is surface runoff. The study domain is a rectangular aquifer zone with length L=35 m and height H=4 m. The initial water table elevation $h_0(x)$ is 1 m, the uniform horizontal space step Δx is 1 m, the uniform vertical space step Δz is 1 cm, and the uniform time step Δt is 0.5 h. In the unsaturated zone, $K(\theta)$ and $D(\theta)$are estimated, based on the Clapp and Hornberger empirical relationships [21], as

$$K(\theta)=K_{s1}\left(\frac{\theta}{\theta_s}\right)^{2b+3} \text{ and } D(\theta)=\frac{-bK_{s1}\psi_s}{\theta_s}\left(\frac{\theta}{\theta_s}\right)^{b+2},$$

respectively, where K_{s1} is the saturated vertical hydraulic conductivity, ψ_s is the saturated water potential, θ_s is the saturated soil moisture, and b is a parameter. The following soil parameters are used: θ_s=0.48, θ_0=0.1594, ψ_s=−200 mm, K_{s1}=0.0063 mm s^{-1}, b=6.0, n_e=0.25.

We first examine the sensitivity of river elevation. Three river elevations are considered: h_r=1.5, 2.5 and 3.5 m. We assume that groundwater horizontal hydraulic conductivity K_s is equal to 0.544 m h^{-1} and zero flux is on the ground surface. It follows from Figure 5(a)–(c) that the elevated water table near river channel increases as river elevation increases. This is because high river elevation can increase hydraulic gradient near the river channel. For the equal groundwater hydraulic conductivity K_s, high hydraulic gradient enhances recharge.

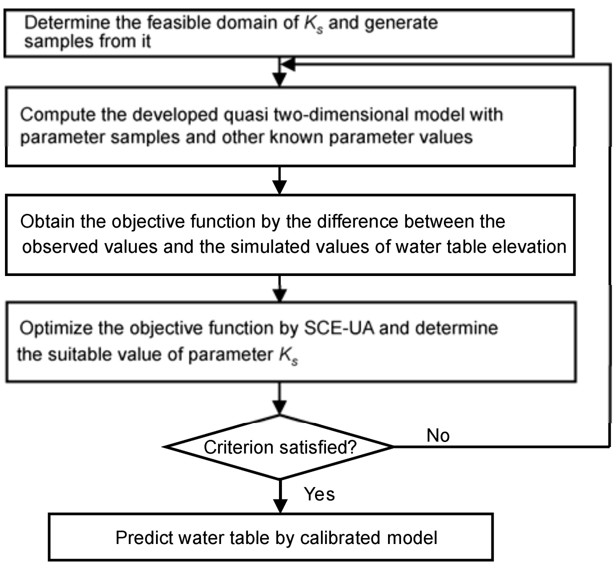

Figure 4 Flowchart for water table prediction based on GSIM and SCE-UA.

We then examine the sensitivity of groundwater horizontal hydraulic conductivity K_s. We take the same parameters values and initial values as the first experiment except for river elevation h_r =3.0 m and groundwater hydraulic conductivity K_s=0.544, 1.088, and 2.176 m h^{-1}, respectively. Zero flux on the surface is assumed in this experiment. It follows from Figure 5(d)–(f) that the differences in water table elevations for gird cells far from the river channel increase as the groundwater hydraulic conductivity increases. This is because the increased groundwater hydraulic conductivity reduces the response time of different grid nodes.

We finally examine the sensitivity of surface flux $P–E–R$. All the parameters values and initial values are same as those described in Figure 5(d) except for surface infiltration flux $P–E–R$ =0.0, 0.1, and 0.15 cm h^{-1}, respectively. It follows from Figure 5(g)–(i) that both soil moisture and water table elevations increase as surface infiltration increases.

2.2 Model validation

In this section, the developed numerical model is validated by an actual case of ecological water conveyance in the lower reaches of the Tarim River. The Tarim River Basin is one of the most severely ecological degradation regions in China. Due to unreasonable usage of water resources and especially inefficient water and land exploitation in the upper and middle breaches, it results in that the river dried up, the water table continuously decreased for lacking of supplies from river, and large areas of natural vegetation died out' in the lower breaches of Tarim River [22, 23]. For this reason, a water conveyance project in the lower reaches of the Tarim River has been implemented since May, 2000. Its purpose is that groundwater on the river sides along a river can be recharged laterally from stream water, and water table can be elevated to a critical depth to make vegetation get enough soil moisture and thus to sustain the ecological balance of riparian zone. The project lasts for 934 days, which are divided into eight phases, and the total discharge is about 21.96×10^9 m^3. There are nine monitoring sections and 40 groundwater-monitoring wells from them for monitoring the dynamic changes of the river discharges and the water table [24, 25].

In Yingsu section, for example, it is the third section of nine sections and 60 km far from the Daxihaizi Reservoir. Yingsu section has seven groundwater-monitoring wells, and we select four monitoring wells (C3–C6) that have detailed data for the studies. The distances between the river

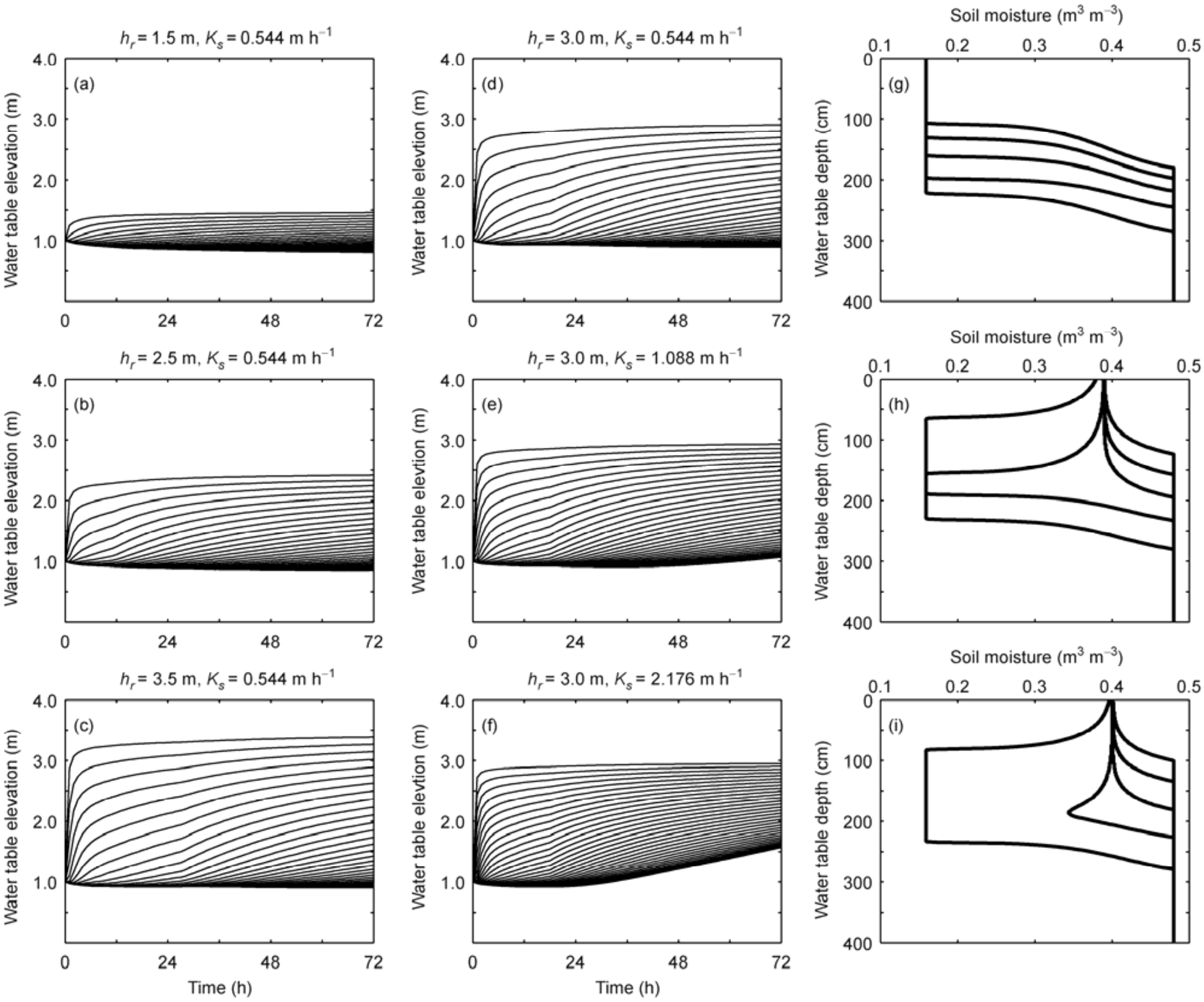

Figure 5 Sensitivities of the river elevation, groundwater hydraulic conductivity, and surface flux. (a)–(c) Water table elevation curves of all level nodes in three days with different fixed river elevation; (d)–(f) water table elevation curves of all level nodes in three days with different groundwater hydraulic conductivity; (g)–(i) are soil moisture in soil column 20 m far from river channel, every 3 days in half a month when the surface flux *P–E–R* is 0.0, 0.1, and 0.15 cm h^{-1}, respectively.

bank and the four monitoring wells are 150, 300, 500, and 750 m, respectively (Figure 6).

The river discharges and the river elevations are observed every day during the period and the phases 5–7, respectively, while the water table elevations are only observed every five days or every month. The data of 81 days (i.e., from 16 November, 2000 to 4 February, 2001) in the second phase of water conveyance are used to calibrate groundwater horizontal hydraulic conductivity K_s of the proposed scheme, while the observed water table elevations of the four monitoring wells (C3–C6)from the third phase to the sixth phase are used to validate numerical simulation results. Two stages have been implemented during the third phase from April to November, 2001, and the total discharge is about 3.81×10^8 m^3. The fourth phase lasts for 110 days from July to November, 2002, and the total discharge is about 2.93×10^8 m^3. Two stages have been also implemented during the fifth phase from March to November, 2003, and the total discharge is about 6.25×10^8 m^3. Similarly, there are two stages during the sixth phase of water conveyance from April to November, 2004, and the total discharge is about 3.5×10^8 m^3.

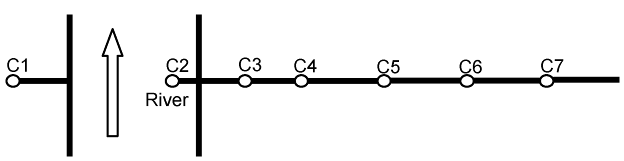

Figure 6 A schematic representation of groundwater-monitoring wells at Yingsu section.

The observed river elevations and discharges are shown in Figure 7. Through regression, the fitted index function can be expressed as

$$H(t) = 832.608 + (Q(t)/5.0354)^{0.4956}. \qquad (22)$$

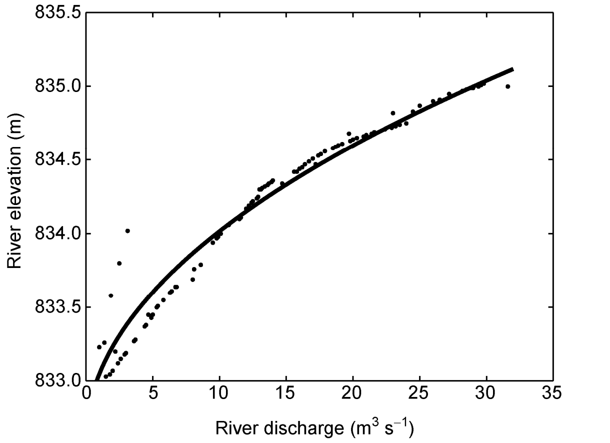

Figure 7 The relation between the observed discharges and river elevations (dotted line) and the fitted rating curve (solid line).

We take the rectangle aquifer with depth 10.2 m from the soil surface and length 1000 m from the right bank as the study domain. The beginning date for integrating the model is 16 November, 2000 in the second phase of water conveyance. The linear regression function for the initial water table elevation is

$$y = -0.0011x + 828.2477\ , \tag{23}$$

where $x[L]$ is the distance (m) from the river bank to the calculation node, and $y[L]$ is the water table elevation (m).

According to the elevations of four monitoring wells, the linear regression function for the ground surface elevation is

$$y(x) = -0.001x + 836.1952\ . \tag{24}$$

where $x[L]$ is the distance (m) from the river bank, and $y[L]$ is the ground surface elevation (m).

For the lower reaches of the Tarim River, there are no available soil texture classification data for numerical model as those developed by Gao et al. [26] for Hei River Basin although soil characteristics have been studied by Yang et al. [27]. Therefore, in accordance with 12 kinds of global soil classification in BATS model [28] and the latitude and longitude of Yingsu section, the sixth kind of sandy clay loam can be taken as θ_s=0.48, θ_0 =0.1594, ψ_s =−200 mm, K_{s1}=0.0063 mm s^{-1}, b=6.0. Let n_e is equal to 0.25. Owing to the uncertainty of soil parameters values acquired from checking table with soil texture classification in larger scale, the groundwater horizontal conductivity K_s, which plays an important role in the estimation of water table depth, needs to be calibrated. We prescribe that the initial soil moisture is 0.4 within the zone of 1 m above water table and 0.2 above the zone of 1 m. The vertical space step is 5 cm, the level space step is 10 m, and the uniform time step is 1 h. Generally E and R should be computed by the land surface model. Because the annual precipitation is less 50 mm [29] and surface runoff is barely formed in this area, we assume that the surface flux P–E–R is equal to zero. With SCE-UA parameter calibration and the developed GSIM, the groundwater horizontal hydraulic conductivity K_s at Yingsu section is determined as 1.588 m d^{-1} in the second phase of water conveyance. Figure 8 shows the observed water table eleva-

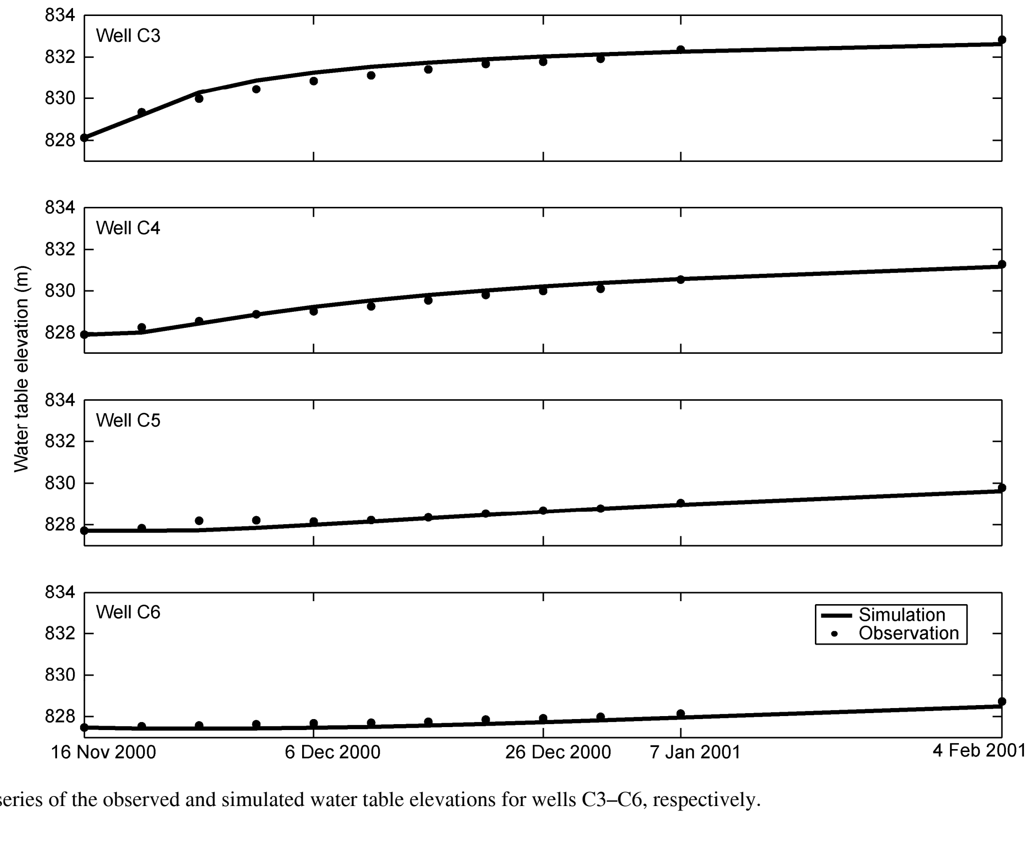

Figure 8 Time series of the observed and simulated water table elevations for wells C3–C6, respectively.

tions (dots) and the estimated water table elevations (lines) for four wells (C3–C6) during the second phase of water conveyance from 16 November, 2000 to 4 February, 2001. It is found that the simulations acceptably agree with observations. The mean absolute error (MAE), root mean squared error (RMSE), and correlation coefficient (CC) are 0.194 m, 0.223 m, and 0.994, respectively.

The numerical model with the calibrated parameter K_s is conducted to predict the spatiotemporal variation of water table elevations from the third phase to the sixth phase at Yingsu section, which is shown in Figure 9. From it, we can see that the simulation results in the third and sixth phase are better than those in the fourth and fifth phase, and the result of well C3 is slightly better than others. Errors of all phases are listed in Table 1, which shows that simulation and observation data have high relativity and the developed model can reasonably simulate variation of water table elevations.

Figure 10 shows the comparison of water table between simulations and observations at Yingsu section on some days during the third and fifth phase. In each subgraph, the four dots denote the observed values in wells C3–C6, respectively, and the solid line denotes the simulated values. The results show that the simulated water table is close to a real case, which demonstrates simplification for two-dimensional moving boundary problem is feasible. Based on the surface elevation, the water table depth can be easily obtained from the solved water table elevation.

3 Conclusions

In this paper, a new quasi two-dimensional framework for water table prediction is presented. We reduce a two-dimensional moving boundary problem of the interaction between soil water and groundwater at river channel cross section to a quasi two-dimensional problem, which consists of a one-dimensional unsaturated vertical soil water flow model, a one-dimensional horizontal groundwater flow model, and an interface model. The quasi two-dimensional

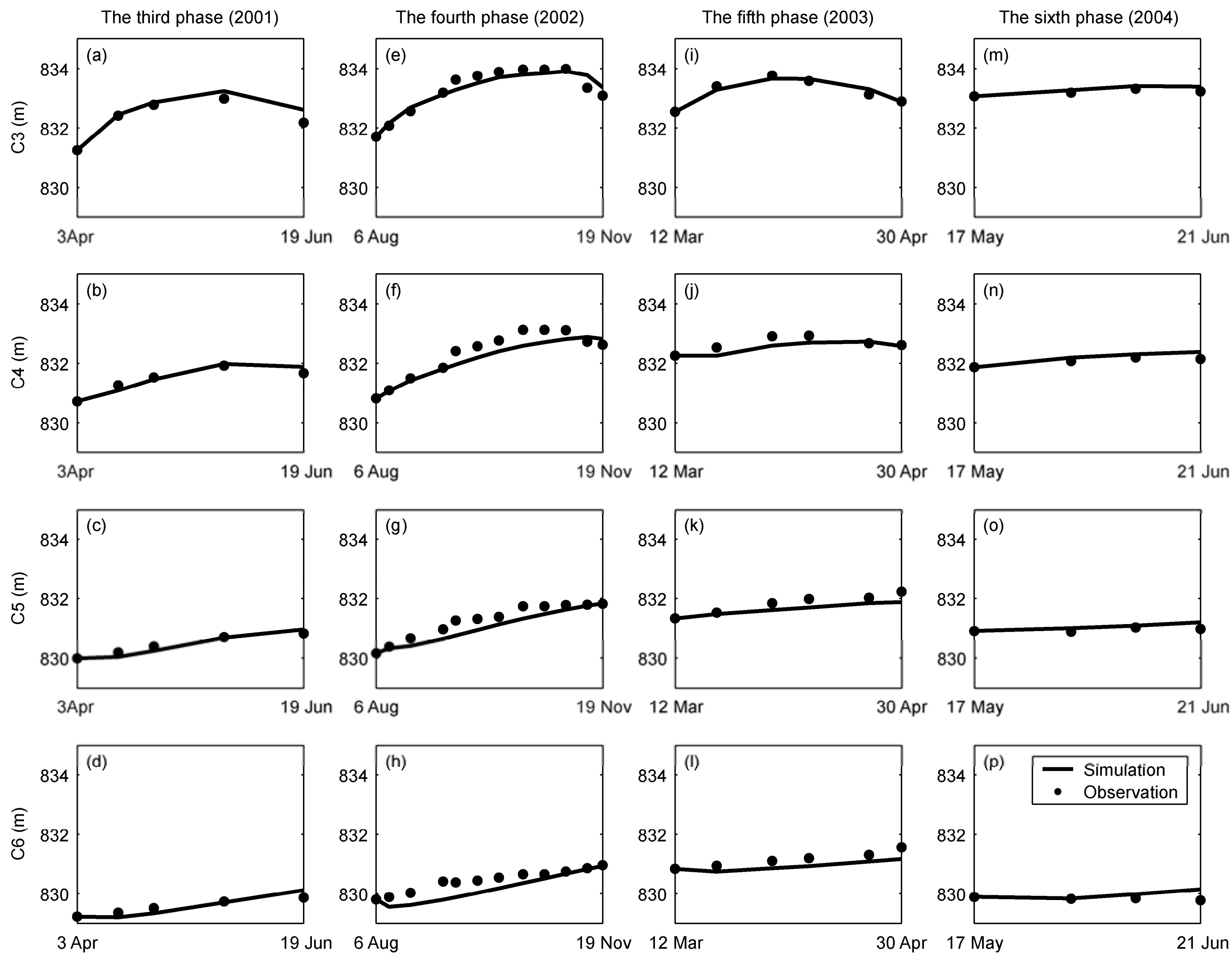

Figure 9 Observed and simulated water table elevations. The rows are for wells C3–C6, and the columns are for the phases of water conveyance from the third to sixth, respectively.

Table 1 Simulation errors for the phase of water conveyance[a)]

Phase	Simulation time (day)	Observation time (day)	MAE	RMSE	CC
2nd	80	12	0.194	0.223	0.994
3rd	77	5	0.119	0.162	0.994
4th	105	12	0.228	0.285	0.985
5th	49	6	0.159	0.200	0.987
6th	35	4	0.107	0.149	0.997

a) MAE is the mean absolute error, RMSE is the root mean squared error, and CC is the correlation coefficient.

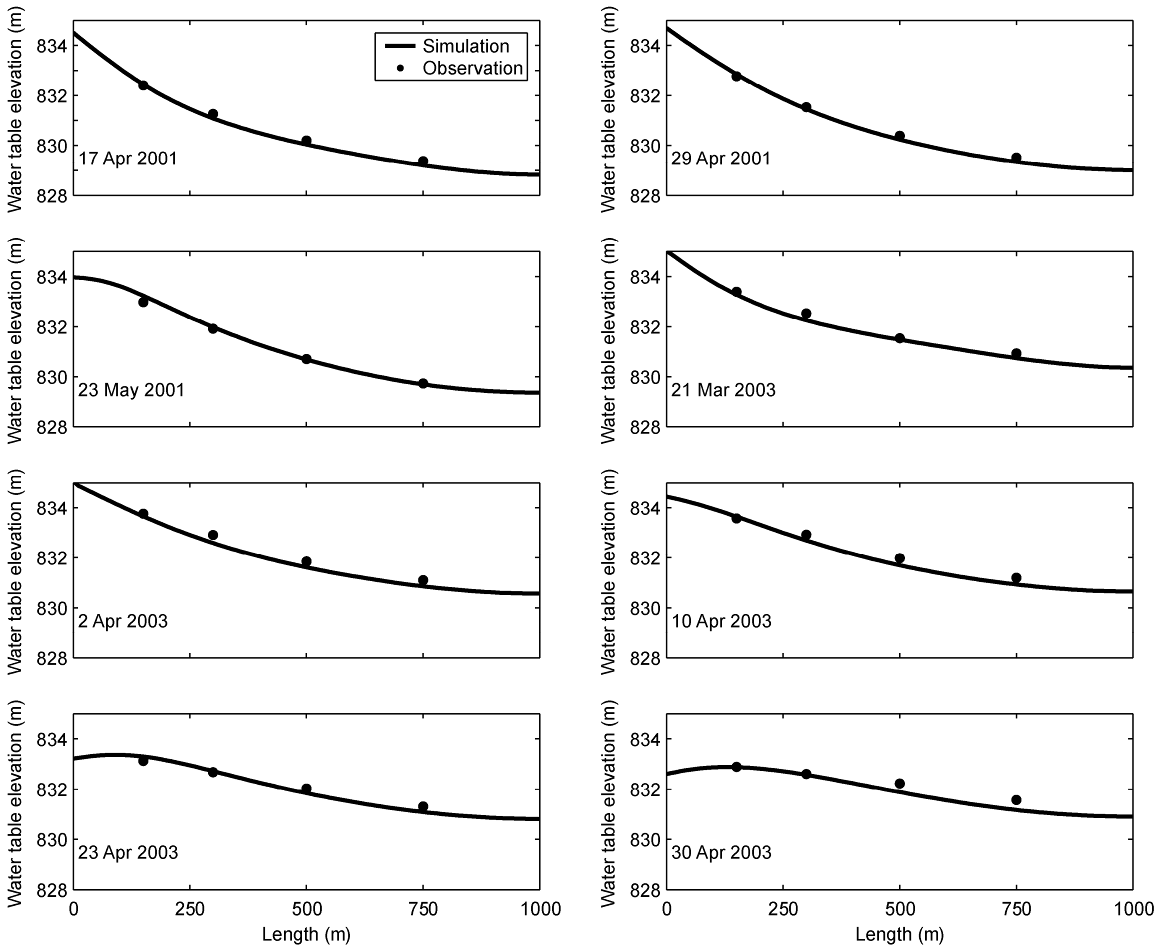

Figure 10 Comparison of the observed and simulated water table at Yingsu section.

model is established to simulate soil moisture and water table depths under the condition of stream water conveyance. The sensitivities of the model parameters (river elevation, horizontal conductivity, and ground surface flux) are tested, which shows the robustness of the proposed model under different conditions. With the calibrated parameter derived from SCE-UA method, the developed quasi two-dimensional model is used to simulated dynamic water table elevation at the Yingsu section with stream water transferred. Simulations are in good agreement with the observations, which shows that the developed model can reasonably simulate dynamic variation of water table and can be used for prediction of water table depths. In this study, the model is with uniform terrain, without consideration of subsurface flow in unsaturated zone, and may be extended to three-dimensional area coupled with current land surface models. However, the developed model needs further study and validation.

This work was supported by National Basic Research Program (Grant Nos. 2010CB428403, 2010CB951001), Chinese COPES Project (Grant No. GYHY200706005), National High Technology Research and Development Program of China (Grant No. 2009AA12Z129).

1 Chen Y N, Li W H, Chen Y P, et al. Water conveyance in dried-up river way and ecological restoration in the lower reaches of Tarim River, China (in Chinese). Acta Geol Sin, 2007, 27: 538–545

2 Ye Z X, Chen Y N, Li W H. The general appraisement of groundwater level after ecological water transportation in the lower reaches of Tarim River (in Chinese). J Arid Land Resour Environ, 2007, 21: 12–16
3 Zhang L H, Chen Y N, Li W H. Analysis on the effect of implementing the project of transfusing stream water for regenerating the ecology in the lower reaches of the Tarim River (in Chinese). Arid Zone Res, 2006, 23: 32–38
4 Wan J H, Chen Y N, Li W H, et al. Variation of groundwater level after ecological water transport in the lower reaches of the Tarim River in recent six years (in Chinese). Arid Land Geogr, 2008, 31: 428–435
5 Xie Z H, Zeng Q C, Dai Y J. et al. Numerical simulation of an unsaturated flow equation. Sci China Ser D-Earth Sci, 1998, 41: 429–436
6 Luo Z D, Xie Z H, Zhu J. Mixed finite element method and numerical simulation for the unsaturated soil water flow problem (in Chinese). J Comput Math, 2003, 25: 113–128
7 Xie Z H, Luo Z D, Zeng Q C, et al. A numerical simulation solving moisture content and flux for an unsaturated soil water flow problem. Prog Nat Sci, 1999, 9: 678–686
8 Yuan X, Xie Z H, Liang M L. Spatiotemporal prediction of shallow water table depths in continental China. Water Resour Res, 2008, 44: W04414, doi: 10.1029/2006WR005453
9 Chen Y N, Zhang X L, Zhu X M, et al. Analysis on the ecological benefits of the stream water conveyance to the dried-up river of the lower reaches of Tarim River, China. Sci China Ser D-Earth Sci, 2004, 47: 1053–1064
10 Xie Z H, Yuan X. Prediction of water table under stream-aquifer interactions over an aired region. Hydrol Process, 2009, 24: 160–169
11 Liang X, Xie Z H, Huang M Y. A new parameterization for surface and groundwater interactions and its impact on water budgets with the variable infiltration capacity (VIC) land surface model. J Geophys Res, 2003, 108: 8613, doi: 10.1029/2002JD003090
12 Liang X, Xie Z H. Important factors in land-atmosphere interactions: Surface runoff generations and interactions between surface and groundwater. Glob Planet Change, 2003, 38: 101–114
13 Yang H W, Xie Z H. A new method to dynamically simulate groundwater table in land surface model VIC. Prog Nat Sci, 2003, 13: 819–825
14 Lei Z D, Yang S X, Xie S Z. Dynamics of Unsaturated Soil Water (in Chinese). Beijing: Tsinghua University Press, 1987. 37
15 Shen Z L, Liu G Y, Yang C T, et al. Hydrogeology (in Chinese). Beijing: Science Press, 1982. 246–247
16 Xue Y Q. Groundwater Dynamics. 2nd ed (in Chinese). Beijing: Geological Publishing House, 1997. 63
17 Bear J. Dynamics of Fluids in Porous Media. New York: American Elsevier Publishing Company, 1972. 764
18 Duan Q Y, Gupta V K, Sorooshian S. Effective and efficient global optimization for conceptual rainfall-runoff model. Water Resour Res, 1992, 28: 1015–1031
19 Duan Q Y, Gupta V K, Sorooshian S. A shuffled complex evolution approach for effective and efficient global minimization. J Optim Theory Appl, 1993, 76: 501–521
20 Duan Q Y, Sorooshian S, Gupta V K. Optimal use of the SCE-UA global optimization method for calibrating watershed models. J Hydrol, 1994, 158: 265–284
21 Clapp R B, Hornberger G M. Empirical equation for some soil hydraulic properties. Water Resour Res, 1978, 14: 601–604
22 Song Y D, Fan Z L, Lei Z D, et al. The Water Resource and Ecology in Tarim River, China (in Chinese). Urumqi: Xinjiang People Press, 2000. 310–332
23 Feng Q, Cheng G D. Towards sustainable development of the environmentally degraded arid rivers of China—A case study from Tarim River. Environ Geol, 2001, 41: 229–238
24 Chen Y N, Cui W C, Zhang Y M, et al. Utilization of water resources and ecological protection in the Tarim River Basin (in Chinese). Acta Geo Sin, 2003, 58: 215–222
25 Xu H L, Chen Y N, Li W H. Study on response of groundwater after ecological water transport in the lower reaches of the Tarim River (in Chinese). Res Environ Sci, 2003, 16: 19–28
26 Gao C Y, Peng H C, Li H Y, et al. Setup and validation of the soil texture type distribution data in the Heihe River Basin (in Chinese). Plateau Meteorol, 2007, 26: 967–974
27 Yang Y H, Chen Y N, Li W H. Soil properties and degree of desertification in lower reaches of Tarim River (in Chinese). J Soil Water Conserv, 2007, 21: 44–49
28 Dickison R E, Henderson-Sellers A, Kennedy P J, et al. Biosphere atmosphere transfer scheme (BATS) for NCAR community climate model. NCAR Technical Note, NCAR/JN 275+STR, 1986
29 Chen Y N, Chen Y P, Xu C C, et al. Effects of ecological water conveyance on groundwater dynamics and riparian vegetation in the lower reaches of Tarim River, China. Hydrol Process, 2009, doi: 10.1002/hyp.7429